OPTIMAL QUANTIZATION IN NON-ORTHOGONAL SUBBAND CODERS*

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ABSTRACT

It is well known that mean square error (MSE) is not preserved under non-orthogonal transformations. This poses a significant challenge to quantize the subband signals in coders that involve non-orthogonal filter banks, since nearest neighbor (NN) encoding rule can no longer be applied to quantize the coefficients. In this paper, we develop techniques for optimal quantization of the subband coefficients in non-orthogonal subband coders. An exhaustive-search based quantization approach is proposed. The complexity of this approach is shown to increase exponentially with the length of the signal. Next, a reduced complexity solution in the form of a trellis-based search is proposed. Simulation results indicate appreciable SNR gains over standard coding techniques for non-orthogonal filter banks.

1. INTRODUCTION

Subband or wavelet based compression techniques are being extensively used in various image compression algorithms. A typical image compression scheme involves a subband decomposition of the image followed by quantization and entropy coding of the subband signals. In order to simplify the quantization of subband signals, it is desired that the analysis and synthesis filter banks are orthogonal. The orthogonality of the filter banks ensures that the mean square error (MSE) between the original and the quantized subband signals is preserved at the output, and hence nearest neighbor encoding rule [1] can be used for quantization of the subband signals. For image compression, an equally desirable characteristic [2] of the analysis and synthesis filter banks is that the filters should be symmetric (or equivalently, they possess the linear-phase property). When two-channel filter banks are used, the two requirements of orthogonality and linear-phase of the analysis and synthesis filters are mututally exclusive [3], [4]. As a result, a number of wavelet based image compression schemes give up the requirement of using orthogonal filter banks and use linear phase (non-orthogonal) filter banks instead. The ability of non-orthogonal filter bank to provide symmetric analysis and synthesis filters is not the only motivation behind their use in subband coding. Recent results [5], [6] have shown that potentially larger coding gains can be achieved by using non-orthogonal (biorthogonal) filter banks. The potential of non-orthogonal filter banks to provide larger coding gains and their inherent flexibility (less constraints) makes them attractive for use in subband coding.

The use of non-orthogonal filter banks, however, complicates the quantization of subband signals. The aim of quantization is no longer to minimize the MSE between the original and quantized subband signal but to minimize the MSE between the input and the reconstructed output signal. This implies that the quantizers used for coding the subband signals need not satisfy the nearest neighbour encoding rule. A number of approaches have been proposed to remedy this problem. Most of the approaches involve making the analysis and the synthesis filters to be as close to orthogonal [8] as possible while satisfying some desirable property (e.g. linear phase). The "quasi-orthogonality" of such filter banks makes nearest neighbor encoding of subband signals, approximately optimal. Another approach is to use a relaxation based quantization procedure [7] which attempts to minimize the MSE between the input and the reconstructed output. However, the relaxation-based quantization procedure, as developed in [7], applies only to the case where a logarithmic division of the frequency axis is used. Another disadvantage of the relaxation-based quantization procedure is that the quantization technique is not necessarily optimal. In this paper, we present techniques for the optimal quantization of the subband signals when a non-orthogonal filter bank or even a Minimum Mean Square Error (MMSE) filter bank [9] is used to do the subband decomposition. Two procedures for optimal quantization in non-orthogonal subband coders are proposed. The complexity of the first scheme is shown to grow exponentially with the length of the input signal while that of the second is shown to increase linearly

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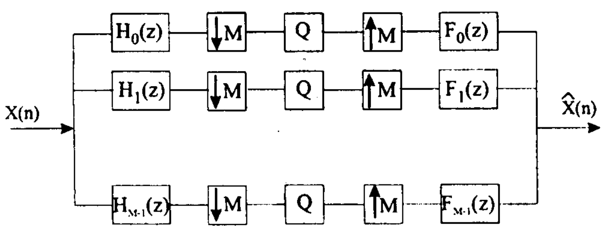

Figure 1: An M-channel subband coder

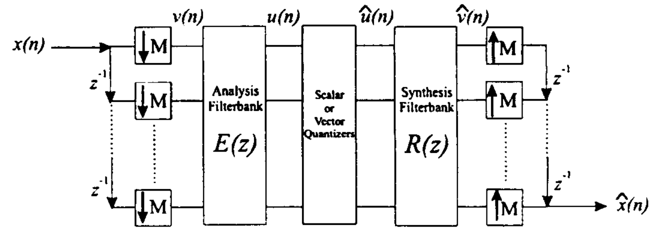

Figure 2: Polyphase Decomposition of the M-channel subband coder

with the signal length. The structure of the optimal non-orthogonal subband coder is shown to be markedly different from traditional subband coders. It is shown that unlike the latter case, analysis filtering and quantization operations are done jointly, by a non-linear encoder, in optimal non-orthogonal subband coders.

2. OPTIMAL QUANTIZATION OF SUBBAND SIGNALS

Consider a typical subband coding scheme as shown in Figure 1. For the sake of conceptual simplicity, we consider an equivalent structure which is obtained by applying the polyphase decomposition to the analysis and synthesis filter bank as shown in Figure 2. In this polyphase structure, the input $x(n)$ is first converted to vectors $\mathbf{v}(n) = [x(Mn), x(Mn-1), \ldots, x(Mn-M+1)]^T$ which are filtered by the block filter $\mathbf{E}(z)$. The subband vectors $\mathbf{u}(n)$ are then quantized either using a scalar quantizer along its various components or by using a vector quantizer (VQ) of dimension M. It is also possible to use intra-band VQ, however such schemes will not be considered here. Without loss of generality, we will consider the case when the vector $\mathbf{u}(n)$ is vector quantized (scalar quantization along individual components can be thought of a VQ whose codebook is obtained as the cross product of the codebooks associated with every scalar quantizer). The subband vector $\mathbf{u}(n)$ is quantized to obtain $\hat{\mathbf{u}}(n)$, which is then filtered through the synthesis block filter $\mathbf{R}(z)$ to produce the output vector $\hat{\mathbf{v}}(n)$. The mean square error (MSE) between the input $x(n)$ and the output $\hat{x}(n)$ is equal to the MSE between the vectors $\mathbf{v}(n)$ and $\hat{\mathbf{v}}(n)$. However, unless $\mathbf{R}(z)$ is orthogonal, the MSE between $\mathbf{u}(n)$ and $\hat{\mathbf{u}}(n)$ may not be equal to the MSE between $\mathbf{v}(n)$ and $\hat{\mathbf{v}}(n)$. For the sake of simplicity, we assume that the length of the input signal, $x(n)$, is a multiple of M i.e. $L = KM$, where L is the length of the input signal. This implies that there are K vectors $\mathbf{u}(n)$ (each of dimension M) in the sequence.

2.1. Exhaustive Search-Based Technique

As the name suggests, this technique involves an exhaustive search through all the possible output sequences that can be produced, and then choosing the one which results in the minimum MSE with the input signal. Assume that the number of code vectors availble for representing the quantized signal, $\hat{\mathbf{u}}(n)$ is N. Then, irrespective of the output of the analysis filters, input to the synthesis filter bank consists of one of the N possible vectors. A simple technique to quantize the subband vectors is as follows – given that the length of the vector sequence $\mathbf{u}(n)$ is K, we construct all the possible N^K reconstructed sequences $\hat{\mathbf{u}}(n)$. Next, each of these sequence is filtered through the synthesis filter bank $\mathbf{R}(z)$ to produce the ouput of the filter bank. The output corresponding to each of the N^K sequence is compared to the input and the particular sequence $\hat{\mathbf{u}}(n)$ which produces minimum MSE is then transmitted to the decoder. A block diagram of the encoder in the proposed scheme is as shown in Figure 3. The decoder is same as that used in traditional subband coders and consists of filtering the quantized sequence through $\mathbf{R}(z)$ and converting $\hat{\mathbf{v}}(n)$ into scalars. This exhaustive search-based quantization procedure requires N^K filtering and comparison operations. In other words, the complexity of the exhaustive search-based quantization procedure grows exponentially with the length of the input signal. Thus, while this quantization scheme is optimal and conceptually simple, the prohibitvely high complexity renders it unattractive for coding signals of even small lengths.

Although the exhaustive search-based quantization technique is impractical, it illustrates a few salient features of the optimal non-orthogonal subband coder. The first feature of this scheme is that, once the codebook of the VQ is fixed, the analysis filters have virtually no role to play in optimal subband coding. This is apparent since the encoding operation involves filtering each of the possible reconstructed sequence through the synthesis filter bank and then choosing the sequence which produces an output sequence with the lowest MSE when compared to the input vector sequence.

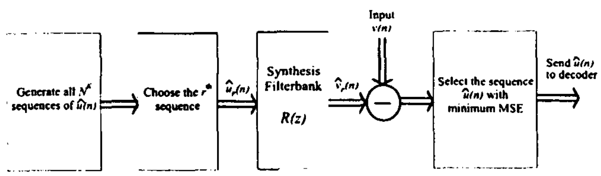

Figure 3: Block diagram of the encoder in an optimal non-orthogonal subband coder

The encoding operation does not make any use of the sequence of vectors $\mathbf{u}(n)$ which are obtained after filtering the input through the analysis filter bank. The second feature of this optimal subband coding scheme is the nonlinear nature of the encoding operation although the synthesis filter bank is linear. Furthermore, the complexity of the scheme is asymmetric in the sense that while the encoder has enormous complexity, only a filtering operation and additions are performed in the decoder. It should be mentioned that the exhaustive search-based technique proposed in this paper is very similar to the analysis-by-synthesis techniques, used frequently in speech coding.

2.2. Trellis-Based Search Technique

The exponential complexity of the exhaustive search-based quantization scheme is the main motivation to develop computationally efficient techniques for quantization in non orthogonal subband coders without any loss in optimality. The key observation that results in a lower complexity quantization scheme is the similarity of the synthesis filter bank to a finite state machine. To illustrate this similarity, let us assume that the order of the block filter $\mathbf{R}(z)$ is P i.e.

$$\mathbf{R}(z) = \sum_{k=0}^{P} \mathbf{R}_k z^{-k}, \tag{1}$$

where $\mathbf{R}_k$ are $M \times M$ matrices. Now the input to the synthesis filter bank, $\hat{\mathbf{u}}(n)$, is a quantized version of the subband signal $\mathbf{u}(n)$, and must hence belong to one of the N possible codevectors. We define the "state" of the synthesis filter bank at any instant of time n as the indices of the past P codevectors, $\hat{\mathbf{u}}(n-1), \hat{\mathbf{u}}(n-2), \ldots, \hat{\mathbf{u}}(n-P)$. Clearly, at any instant, the number of possible states that can be associated with the synthesis filter bank are N^P. The output of the synthesis filter bank, $\hat{\mathbf{v}}(n) = \sum_{k=0}^{P} \mathbf{R}_k \hat{\mathbf{u}}(n-k)$, can be rewritten as

$$\hat{\mathbf{v}}(n) = \hat{\mathbf{v}}_s(n) + \hat{\mathbf{v}}_c(n), \tag{2}$$

where

$$\hat{\mathbf{v}}_c(n) = \mathbf{R}_0 \hat{\mathbf{u}}(n) \quad \text{and} \quad \hat{\mathbf{v}}_s(n) = \sum_{i=1}^{P} \mathbf{R}_i \hat{\mathbf{u}}(n-i). \tag{3}$$

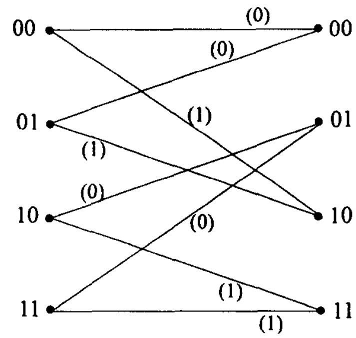

Figure 4: State transitions of a trellis corresponding to a second-order $\mathbf{R}(z)$ with two reconstruction codevectors in the VQ codebook. The states of the trellis are labeled by the indices of the coodevectors in the synthesis filter bank memory. The numbers in the parenthesis on the branches indicate the index of the current reconstruction vector

Eq.(3) illustrates the fact that the output of the synthesis filter bank is composed of two terms – $\hat{\mathbf{v}}_s(n)$ which depends entirely upon the state of the filter bank and $\hat{\mathbf{v}}_c(n)$ which depends only upon the choice of the current reconstruction vector. Furthermore, the choice of a particular reconstruction vector, $\hat{\mathbf{u}}(n)$, determines the state of the synthesis filter bank at time $n+1$. There are N possible state transitions from a particular state at time n to the states at time $n+1$. These state transitions can be represented by means of a trellis, a part of which is shown in Figure 4 for the case when $\mathbf{R}(z)$ is a second-order block filter and when $N = 2$. The state of the filter bank at time n uniquely identifies the indices of the past P codevectors. Thus, the problem of finding the sequence of optimal reconstruction vectors $\hat{\mathbf{u}}(n)$ is equivalent to finding the sequence of states in the trellis that result in lowest distortion. Furthermore, the MSE after time $n+1$ depends only upon the state of the synthesis filter bank at $n+1$ and not upon the path used to reach the state at $n+1$. This implies that the Viterbi Algorithm [11] can be used to efficiently carry out the search for minimum distortion path. The resulting algorithm for quantization of subband signals is optimal and computationally efficient owing to the use of Viterbi Algorithm. The use of Viterbi Algorithm allows us to efficiently discard a large number of sequences (out of the total N^K sequences) which have been proved to have a larger distortion than the optimal sequence, resulting in an efficient quantization scheme.

As noted in the case of the exhaustive search-based quantization technique, the analysis filters play no role in a subband coder that employs a trellis-based search to find the optimal sequence of reconstruction codevectors. The processes of filtering and quantization are done jointly by a

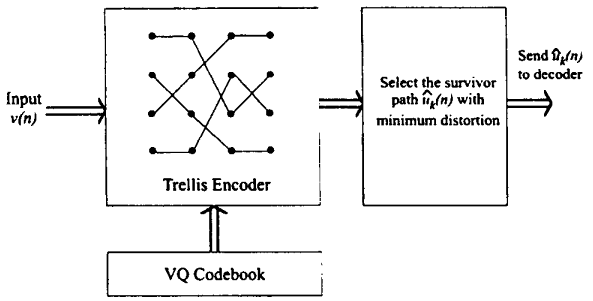

Figure 5: Block diagram of the encoder in a trellis-based subband coder

non-linear trellis encoder. Figure 5 shows the block diagram of a subband coder which employs a trellis based search for the optimum sequence of reconstruction vectors. The complexity of the trellis-based quantization technique is proportional to $K \times N^{P+1}$. The complexity of the quantization algorithm is thus, exponential in the order of the synthesis block filter but is only linear in the length of the signal. The complexity of the trellis-based quantization procedure can be reduced further by pre-computing the values of $\hat{\mathbf{v}}_s(n)$ corresponding to each of the N^P states and also the values of $\hat{\mathbf{v}}_c(n)$ corresponding to each of the N codevectors. In this case, the filtering operation involves just adding pre-computed vectors. It is also possible to reduce the complexity of the trellis-based approach, at the cost of optimality, by using reduced state search [1] techniques.

3. RESULTS

The performance of the proposed trellis-based quantization scheme was compared to that of conventional non-orthogonal subband coders that employ nearest neighbor encoding rule for quantization of the subband signals. For this purpose 1024 samples of an AR(1) source with $\rho = 0.95$ were encoded at 1.0 and 2.0 bits per sample by a two-channel subband coder. The analysis and synthesis filters that were used for comparisons included the 9/7 filters used in [10], the 9/3 filters used in [7], the 11/5 filters used in [7] and the 3/5 filter in [4]. The reconstruction codevectors were generated by using the Generalized Lloyd Algorithm (GLA) over a set of training vectors. The training vectors were generated by filtering an AR(1) signal (different than the test signal) through the analysis filter bank. Same codevectors were used in both the trellis-based quantization scheme and the conventional subband coding scheme. The results are shown in Table 1 which compares the signal-to-noise (SNR) at the output of the subband coders in both cases. Our results indicate that the optimal trellis-based quantization scheme out performs the sub-optimal nearest neighbor encoding schemes by upto 2 dB for some non-orthogonal filter banks. In the trellis based search we need to transmit the state in which the decoder should start filtering. This results in an overhead of NP bits (N is the number of vectors in the codebook and P is the order of the synthesis filter bank filter). Typically, the overhead involved is small and the percentage increase in bit-rate is marginal when the input sequence is of large length. Alternatively, we can avoid the overhead by constraining the trellis search to always begin from a particular pre-determined state.

Note that it is possible to obtain better results for the trellis

Filter Bank used	$b = 1.0$ bits/sample	
	SNR^1 in dB	SNR^2 in dB
9/7	7.42	7.45 (+0.03)
9/3	7.91	7.98 (+0.07)
11/5	9.18	9.65 (+0.47)
3/5	5.56	6.15 (+0.49)
	$b = 2.0$ bits/sample	
9/7	12.53	12.86 (+0.33)
9/3	12.7	13.15 (+0.45)
11/5	12.88	14.37 (+1.49)
3/5	10.53	12.9 (+2.37)

Table 1: The column under SNR^1 shows the SNR in dB at the output of the conventional non-orthogonal subband coders whereas the column under SNR^2 shows the SNR in dB at the output of the trellis based non-orthogonal subband coder for an AR(1) process for different bit-rates. The numbers in the bracket indicate the SNR gains in dB obtained by the optimal trellis based subband coding scheme.

based quantization approach by designing the codebook tailored to the algorithm. However, the main aim of the current simulations were to demonstrate the gains obtained primarily by the algorithm itself.

4. CONCLUSIONS

In this paper, we proposed two schemes for optimal quantization in subband coders when non-orthogonal filter banks are used. The main difference between the two schemes is the computational complexity associated with each scheme. While the complexity of the exhaustive search-based quantization scheme grows exponentially with the length of the input signal, the complexity of the trellis-based scheme grows linearly. Both schemes involve a nonlinear encoder that performs joint filtering and quantization of the input signal unlike a traditional subband coder where the filtering and quantization operation are performed separately.

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