A COMPUTATIONALLY EFFICIENT DESIGN OF TWO-BAND QMF BANKS BASED ON THE FREQUENCY SAMPLING APPROACH*

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ABSTRACT

Two new methods for computationally efficient design of two-channel quadrature mirror filter (QMF) banks based on the frequency sampling approach are introduced. In the proposed approach, the number of parameters to be optimized are reduced significantly, thereby leading to a faster design of the two-channel QMF banks. The characteristics of the filters obtained are comparable to those of some of the existing QMF banks, in terms of the overall amplitude distortion and the minimum stopband attenuation.

1. INTRODUCTION

One-dimensional quadrature mirror filter (QMF) banks have been used extensively in the subband coding of speech and images [1, 2]. A typical two-band filter bank is shown in Figure 1. In a two-band QMF bank the analysis filters are related as $H_1(z) = H_0(-z)$. To ensure aliasing cancelation, the synthesis filters satisfy the constraint $G_0(z) = H_0(z)$ and $G_1(z) = -H_0(-z)$. If $H_0(z)$ is designed to have linear phase, then, the resulting distortion function, given by $T(z) = \frac{1}{2}(H_0^2(z) - H_0^2(-z))$ has linear phase, thereby eliminating phase distortion. The residual amplitude distortion can be minimized by an appropriate design of $H_0(z)$.
Although there exist methods for the design of two-band perfect reconstruction filter banks, the associated analysis and synthesis filters (FIR) may not have as good a magnitude response as that of filters in a QMF bank. Analysis and synthesis filters with good magnitude response are required to remove out-of-band quantization noise and correlation between the two channels. Moreover, due to the linear phase property as well as the constraint $H_1(z) = H_0(-z)$, the QMF banks have low implementation complexity in terms of the number of multipliers required. As a result, QMF banks are attractive for use in certain applications.

*This work was supported by a University of California MICRO grant with matching support from Envision Medical Corporartion and Xerox Corporation.

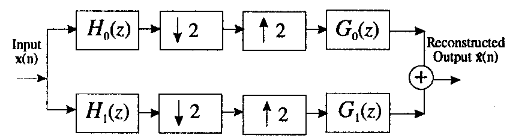

Figure 1: A two-band filter bank

2. DESIGN OF TWO-CHANNEL QMF BANKS

A general objective function for the total cost in the design of a two-channel QMF bank can be formulated as [3]

$$\phi = \alpha E_1 + (1-\alpha)E_2, \tag{1}$$

where

$$E_1 = \int_{\omega_s}^{\pi} |\, H_0(e^{j\omega}) \,|^2 \, d\omega, \qquad \text{and}$$

$$E_2 = \int_0^{\pi/2} \left(1 - |\, H_0(e^{j\omega}) \,|^2 - |\, H_0(e^{j(\pi-\omega)}) \,|^2\right)^2 d\omega.$$

The above cost function can be interpreted as a minimization of the residual amplitude distortion, as given by E_2 and the stopband attenuation of the filter $H_0(z)$, as given by E_1. The cost function ϕ can be minimized by optimizing on the impulse response coefficients [3]. The difficulty associated with such a technique is that optimization has to be performed over a large number of parameters, which may be as many as half the number of impulse response coefficients. As a result, the design complexity for such a method is very high. This is undesirable in applications where the characteristics of the filters (e.g. transition region) have to be changed

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frequently. Furthermore, if the optimization is performed directly over the impulse response coefficients, it is difficult to find reasonable initial values for these coefficients. In order to reduce the design complexity, we propose, in this paper, two methods for the design of two-band QMF banks based on frequency sampling approach. The primary advantage of these techniques is that they yield results comparable to those in [3] without requiring optimization over a large number of parameters. Earlier work [4, 5] on reducing the design complexity of QMF banks have focussed on converting the cost function given by Eq. (1) to a quadratic form.

3. FREQUENCY SAMPLING DESIGN OF TWO-BAND QMF BANKS

3.1. First Method

In order to determine the impulse response coefficients of $H_0(z)$, we determine the magnitude of samples of $H_0(e^{j\omega})$ at $\omega = 2\pi k/L$, where L is the length of the filter. An inverse DFT of the frequency samples then yields the desired impulse response coefficients [6]. The constraints on the magnitude response $|H_0(e^{j\omega})|$ of the lowpass analysis filter are as follows: it should be approximately 1 in the passband, close to 0 in the stopband and should also satisfy the power complementary relation

$$\mid H_0(e^{j\omega}) \mid^2 + \mid H_0(e^{j(\pi-\omega)}) \mid^2 = 1. \qquad (2)$$

If $|H_0(e^{j\omega})|$ is equal to 1 in the passband and equal to 0 in the stopband, then Eq.(2) is satisfied in both regions. This can be approximately achieved by fixing the magnitude of the samples, $m[k]$ of $H_0(e^{j\omega})$, equal to one in the passband and zero in the stopband respectively. Now $H_0(e^{j\omega})$ can be written as [6]

$$H_0(e^{j\omega}) = \frac{e^{-j\omega(L-1)/2}}{L} \sum_{k=0}^{L-1} \frac{H(k)e^{-j\pi k/L} sin(\omega L/2)}{sin(\omega/2 - \pi k/L)}$$

where

$$H(k) = \begin{cases} m[k]\phi_1(k) & k = 0, \ldots, L/2-1, \\ 0 & k = L/2 \\ m[L-k]\phi_1^*(L-k) & k = L/2+1, .., L-1. \end{cases}$$

and $\phi_1(k) = e^{(-j\pi k(L-1)/L)}$.
Let

$$\mathbf{m} \;=\; [m[0] \;\; m[1] \; \ldots \; m[L-1]\,]^T\,, \qquad (3)$$

and let $\mathbf{f}(\omega)$ be a column vector of length L whose k^{th} element is given by

$$f[k] = \frac{sin(\omega L/2)}{sin(\omega/2 - \pi k/L)}.$$

Then, the amplitude distortion and the stopband energy of $H_0(e^{(j\omega)})$ can be written in terms of $\mathbf{m}$ and $\mathbf{f}(\omega)$ as

$$\begin{aligned}(\mid H_0(e^{j\omega}) \mid^2 + \mid H_0(e^{j(\pi-\omega)}) \mid^2 - 1)^2 = \\ ((\mathbf{m}^T\mathbf{f}(\omega))^2 + (\mathbf{m}^T\mathbf{f}(\pi-\omega))^2 - 1)^2 = \\ (\mathbf{m}^T\mathbf{Q}(\omega)\mathbf{m} - 1)^2,\end{aligned} \qquad (4)$$

and

$$\mid H_0(e^{j\omega}) \mid^2 = \mathbf{m}^T\mathbf{f}(\omega)\mathbf{f}^T(\omega)\mathbf{m},$$

where $\mathbf{Q}(\omega) = \mathbf{f}(\omega)\mathbf{f}^T(\omega) + \mathbf{f}(\pi-\omega)\mathbf{f}^T(\pi-\omega)$. To design the QMF bank, we need to find $\mathbf{m}$ such that the cost function given by Eq. (1) is minimized. Since all the components of $\mathbf{m}$ that occur in the passband and stopband have already been set to 1 and 0, respectively, we are left to find values of only those components which occur in the transition band. This can be done by choosing their values, such that an appropriate cost function is minimized. Thus, the optimization problem can be reformulated as follows: determine the unknown components of $\mathbf{m}$ such that

$$\begin{aligned}\phi = \alpha \int_0^{\pi/2} (\mathbf{m}^T\mathbf{Q}(\omega)\mathbf{m} - 1)^2 d\omega \;\; + \\ (1-\alpha)\int_{\omega_s}^{\pi} \mathbf{m}^T\mathbf{f}(\omega)\mathbf{f}^T(\omega)\mathbf{m} d\omega\end{aligned} \qquad (5)$$

is minimized. This optimization is performed only over the transition band samples, which are usually small in number as compared to the filter length. Thus, the optimization complexity is not very high. Once the frequency samples are obtained in this way, an inverse DFT yields the impulse response coefficients of $H_0(z)$. The only unspecified quantity is the width of the transition band, which could be one of the input parameters to the algorithm. The width of the transition band controls the number of samples which occur in the transition band, and thereby controls the complexity of the optimization.

The key step in this design procedure is fixing the magnitudes of samples in the passband to be equal to one, and fixing the magnitudes of the samples in the stopband to be equal to zero. While these may not be the optimal values of the frequency samples, they are very close to the optimal values for filters with good magnitude response. Simulation results show that loss in optimality is not very significant by fixing these samples. However, a significant advantage of not using these samples for optimization is that it leads to a considerable reduction in the complexity of the design procedure.

3.2. Second Method

The complexity of the iterative optimization is reduced in the first method by setting the magnitude of the frequency samples that occur in the passband and stopband to be equal to 1 and 0, respectively. If the magnitude of the samples falling in the transition band can be specified in a similar way, then, no optimization would be required. In this context, consider an analytical function for the magnitude response of $H_0(e^{j\omega})$:

$$H_0(e^{j\omega}) = \begin{cases} 1 & \text{if } 0 \leq \omega < \omega_p \\ H_t(\omega) & \text{if } \omega_p \leq \omega \leq \omega_s \\ 0 & \text{if } \omega_s < \omega < \pi. \end{cases} \tag{6}$$

The transition region magnitude function $H_t(\omega)$ should satisfy the following properties in order that $H_0(e^{j\omega})$ possesses a continuous magnitude response :

$$H_t(\omega) = \begin{cases} 1 & \text{at } \omega = \omega_p \\ 0 & \text{at } \omega = \omega_s. \end{cases}$$

In addition, to minimize the amplitude distortion, $H_t(\omega)$ should also satisfy

$$H_t^2(\omega) + H_t^2(\pi - \omega) = 1. \tag{7}$$

There exist an infinte number of functions which satisy Eq. (7). However, in this paper, we will consider only those functions that can be expressed as polynomials in ω. In order to have a smooth transition region it is desired that the derivatives of the function $H_t(\omega)$ be equal to zero at $\omega = \omega_p$ and ω_s. It can be established that the general form of such polynomials which have the first m derivatives equal to zero at $\omega = \omega_p$ and ω_s, and also satisfy Eq. (7), is given by

$$H_t(\omega) = \left(x^{m+1} \sum_{k=0}^{k=m} \frac{(m+k)!}{k!m!}(1-x)^k \right)^{\frac{1}{2}}, \tag{8}$$

where $x = (\frac{\omega_s - \omega}{\omega_s - \omega_p})$.

This completely characterizes the magnitude response of the two-band QMF bank. This magnitude response is uniformly sampled at the specified number of points, and then an inverse DFT is performed on these sample values to obtain the impulse response coefficients. Although the frequency sampling technique only guarantees that Eqs. (6) and (7) are satisfied exactly at the sampling frequencies, the deviation from the desired values is small throughout the frequency range. This is due to the fact that the function to be approximated is smooth. Since only an approximation to the desired response can be obtained, the resulting QMF bank exhibits amplitude distortion.

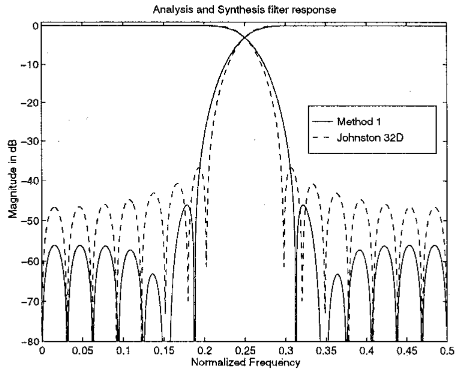
4. RESULTS

The above methods have been used to design linear phase analysis/synthesis filters of length 32. The only other parameter required by the algorithm is the width of the transition region. The value of α is chosen to be equal to one, and a transition width of 0.18π is used for both methods. The value of m in Eq. (8) is chosen to be equal to 2 for the second method. The magnitude response of the filters and the resulting amplitude distortion for the first and second methods, as well as those of Johnston's 32D filters are plotted in Figures 2 and 3, respectively. The results indicate that the first method produces filters of good magnitude response and low amplitude distortion (peak distortion of 0.015dB). Thus, although some of the parameters are set to fixed values, the effect of fixing them is small on the magnitude response of the filters and the overall distortion of the QMF bank.

The second method also leads to filters with lower amplitude distortion (peak value of 0.02 dB) than Johnston's QMF banks but with a slightly degraded magnitude response in terms of larger transition width. The low amplitude distortion verifies the fact that the deviation from the desired response is small throughout the frequency range. The principal advantage of the second method is that it does not require an optimization procedure for the design of the QMF banks.

5. CONCLUDING REMARKS

In this paper, we have developed two approaches, based on the frequency sampling technique, for the design of two-band QMF banks. The main objective of both approaches is to reduce the design complexity of two-band QMF banks. The first method achieves this by reducing the number of parameters involved in the optimization to about 2 to 3 (the typical number of samples occuring in the transition band), while the second method eliminates the need for optimization in the design of filter banks. The second method achieves this by formulating a smooth analytical expression for the desired magnitude response of the filters, and then using a frequency sampling technique to approximate the magnitude response. Since the design involves no optimization, the procedure is extremely fast. The design methods are not aimed at synthesizing filters to meet a specified minimum stopband attenuation or have minimum amplitude distortion. The emphasis rather is to develop fast design methods which result in filters with characteristics comparable to some of the existing filters.



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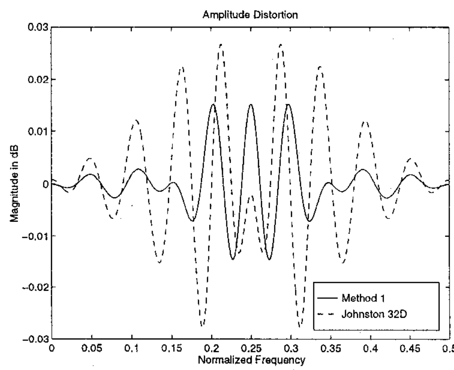
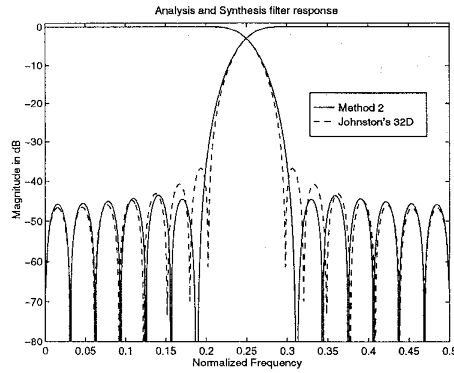


(b)

Figure 2: (a) Magnitude response and (b) Residual amplitude distortion, for filters of length 32 designed using the first method.



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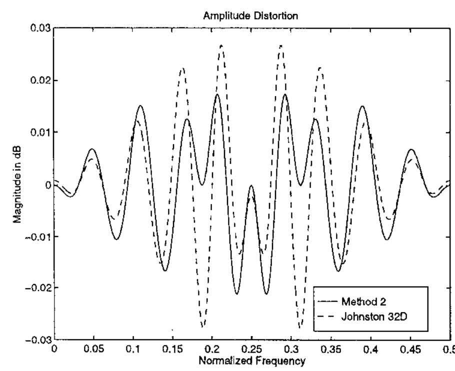

(b)

Figure 3: (a) Magnitude response and (b) Residual amplitude distortion, for filters of length 32 designed using the second method.

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