IMAGE SEGMENTATION USING CURVE EVOLUTION AND REGION STABILITY

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ABSTRACT

A novel scheme for image segmentation is presented. An image segmentation criterion is proposed that gathers similar pixels together to form regions and creates boundaries between two dissimilar regions. This criterion is formulated as a cost function. This cost function is minimized by using gradient-descent methods, which leads to a curve evolution equation that segments the image. The proposed method generalizes previous methods to more complex similarity and distance measures and can be applied to vector valued images such as texture and color images.

1. INTRODUCTION

Image segmentation is a basic step in many image processing and computer vision tasks. Previous approaches to image segmentation include filtering-based methods to detect edges followed by edge linking, curve evolution and active contour models [1,5,6,7,8], region growing and merging, global optimization based on energy functions and Bayesian criteria, and graph partitioning and clustering. Some of these methods seek to provide a unified framework that enables segmentation based on multiple heterogeneous attributes such as texture, color, and gray level intensity.

Curve evolution methods have been used for image segmentation for over a decade. Some of these methods utilize the geometric nature of the curves to evolve them, and some of them use a cost function to guide the curve evolution. In this paper we define an image segmentation criterion and formulate it as a cost function. This cost function is then used to guide the curve evolution to segment the image into homogenous and distinct regions.

In comparison to the previous work [5,7], where simple statistics such as intensity mean and standard deviation are utilized, the proposed method lays a generalized framework where the previous methods can be thought as special cases. This flexible framework accepts wide variety of features for similarity of pixels, such as their intensity, texture and color features as well as the distance between the pixels.

The rest of the paper is organized as follows. We review curve evolution methods in section 2. In section 3, we present a region-based approach to segmentation using geometric active contours. In section 4 we present some experimental results and conclude with discussions in section 5.

2. PREVIOUS WORK

Active contours and curve evolution methods usually define an initial contour C_0 and deform it towards the object boundary. The problem is usually formulated using partial differential equations (PDE). The previous research follows two different paths in terms of representation and implementation of active contours, namely parametric active contours (PACs) and geometric active contours (GACs). PACs use a parametric representation of the curves and GACs utilize level set methods [2,3]. Level set methods can easily handle topology changes of the evolving contour such as splitting and merging, and singularities on the curve such as sharp corners. Recently some connections between these two methods have been established [1, 4]. A summary and comparison of both GACs and PACs can be found in [4].

Curve evolution methods can be classified into several groups: edge-based [1], region-based [5,6,7] and hybrid [8] active contours. Our implementation in this paper is based on region-based GAC methods.

Region-based active contour methods attempt to partition the image into two regions: foreground and background. They start with an initial closed contour and modify the curve according to the statistics of the interior and exterior of this contour. Region-based methods use global image features as opposed to the local features used in edge-based methods.

Developments in region-based active contours [9] are more recent than their edge-based counterparts. Region-based active contours are less dependent on the initial location of the contour since they don't rely much on the local image features. Also not needing to use the gradient of the image simplifies both the variational formulation and its solution. Region-based methods are also easier to extend to vector valued images such as color and texture images.

Let $C(\varphi):[0,1] \to \Re^2$ be a parameterization of a 2-D closed curve, I be a function defined on a closed region R, R_i and R_o be the interior and the exterior of C, m_i and m_o be the corresponding means, A_i and A_o the areas of R_i and R_o respectively, I_i be $I(R_i)$ defined on R_i and I_o be $I(R_o)$ defined on R_o. Tsai, et al. [5] define their optimization criteria as maximizing the separation of the

mean values: $(m_i - m_o)^2$. This leads to a gradient-descent equation

$$\frac{\partial C}{\partial t} = (m_i - m_o)\left(\frac{I - m_i}{A_i} + \frac{I - m_o}{A_o}\right)\vec{N} - \gamma\kappa\vec{N} \quad (1)$$

where $\vec{N}$ is the normal vector to C, κ is the curvature and γ is a constant weighting factor. The κ dependent term is added to keep the curve smooth at all times.

Chan, et al. [7] on the other hand uses a limiting version of Mumford-Shah functional [10] as the criteria, where the image is modeled with a piecewise constant functions. The resultant gradient-descent equation is:

$$\frac{\partial C}{\partial t} = (m_i - m_o)(I - m_i + I - m_o)\vec{N} - \gamma\kappa\vec{N} \quad (2)$$

Later on Tsai et al. [6] generalized this equation by solving the general Mumford-Shah problem instead of the limiting case.

3. PROPOSED IMAGE SEGMENTATION

In this section, our aim is to define a new flow equation for image segmentation that satisfies the following objectives:

1) The evolving curve segments the image into homogenous regions. Homogenous region means that the pixels in this region that are close to each other have similar properties such as their intensity. This criterion for the segmentation groups similar pixels together into regions.

2) The evolving curve establishes boundaries between regions that are distinct in term of their content, which means, pixels corresponding to neighboring regions are not similar to each other. This criterion aims to increase the dissimilarity across the boundaries.

Let C_0 be a 2-D closed curve partitioning an image R into two regions: the interior R_i and the exterior R_o of the curve. Let the image I be a continuous function defined on R. Let $w(p_i, p_j)$ be a positive, symmetric function, which is a measure for the dissimilarity between points p_i and p_j, $p_i, p_j \in R$. Examples for $w(p_i, p_j)$ are

$$w(p_i, p_j) = |I(p_i) - I(p_j)| \quad (3)$$

$$w(p_i, p_j) = |I(p_i) - I(p_j)|e^{\frac{-dist(p_i, p_j)}{\sigma_{dist}}} \quad (4)$$

where $dist(p_i, p_j)$ is either L1 or L2 distance.

To satisfy the first criterion C_0 needs to be evolved in a way to maximize intra pixel similarities for both R_i and R_o. To achieve this, a cost function is needed to guide the curve evolution. Assuming that the image only consists of a foreground object and a background, minimizing the energy functional

$$E_1 = \iint_{R_i\,R_i} w(s_1, s_2)ds_1 ds_2 + \iint_{R_o\,R_o} w(s_1, s_2)ds_1 ds_2 \quad (5)$$

defines the desired first criterion.

On the other hand, to be able to satisfy the second criterion, the curve evolution should also increase the dissimilarity of the pixels of R_i and R_o. This corresponds to minimizing another energy functional

$$E_2 = -2\iint_{R_i\,R_o} w(s_1, s_2)ds_1 ds_2 \quad (6)$$

Since it is difficult to find the direct solutions for these energy minimization problems, gradient-descent methods are usually utilized.

Lemma 1: Let C_1 and C_2 be arbitrary closed curves defined on R, $w: \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$ be a real valued function,

$$E = \int_{R(C_1)} \int_{R(C_2)} w(s_1, s_2)ds_1 ds_2 \quad (7)$$

be the cost functional to be minimized by evolving C_1 and C_2. The notation $R(C)$ corresponds to the area inside the curve C. The first variation and the corresponding gradient descent of this energy functional are as following.

$$\frac{\partial E}{\partial t} = \int_{C_1}\left\langle \frac{\partial C_1}{\partial t}, \left(\int_{R(C_2)} w(c_1, s)ds\right)\vec{N}_1\right\rangle + \int_{C_2}\left\langle \frac{\partial C_2}{\partial t}, \left(\int_{R(C_1)} w(c_2, s)ds\right)\vec{N}_2\right\rangle$$

$$\Rightarrow \frac{\partial C_1}{\partial t} = -\int_{R(C_2)} w(c_1, s)ds\,\vec{N}_1, \quad \frac{\partial C_2}{\partial t} = -\int_{R(C_1)} w(c_2, s)ds\,\vec{N}_2$$

where $\vec{N}_1$ and $\vec{N}_2$ are the outwards normal vectors for C_1 and C_2 respectively, $c_1 \in C_1$ and $c_2 \in C_2$.

Proof: A full proof is out of the scope of this paper because of space restrictions. Intuitively the results can be achieved as following. From previous work [11], we know that the first variation and the gradient-descent for the functional $K = \int_{R(C)} H(s)ds$ is

$$\frac{\partial K}{\partial t} = \int\left\langle C_t, H\vec{N}\right\rangle \quad \Rightarrow \quad \frac{\partial C}{\partial t} = -H\vec{N}$$

By taking $H = \int_{R(C)} w(c, s)ds$, where $C = C_2$ and $C = C_1$ the results in Lemma 1 follow. □

Corollary 1: If $C = C_1 = C_2$ in (7), which means $\vec{N} = \vec{N}_1 = \vec{N}_2$, then

$$\frac{\partial C}{\partial t} = -2\int_{R_i} w(c, s)ds\vec{N}$$

Corollary 2: If $C = C_1 = -C_2$ in (7), which means $\vec{N} = \vec{N}_1 = -\vec{N}_2$, then

$$\frac{\partial C}{\partial t} = \left(\int_{R_o} w(c, s)ds - \int_{R_i} w(c, s)ds\right)\vec{N} \quad (8)$$

Using corollaries 1 and 2, we notice that both (5) and (6) result the same gradient descent equation. This is not surprising considering the relation

$$E_1 = E_2 + \underbrace{\iint\limits_{R\,R} w(s_1, s_2) ds_1 ds_2}_{\text{A Constant}} = E_2 + k \qquad (9)$$

where k is a constant. The gradient-descent flow corresponding to both E_1 and E_2 is

$$\frac{\partial C}{\partial t} = 2 \left(\int\limits_{R_o} w(c,s) ds - \int\limits_{R_i} w(c,s) ds \right) \vec{N} \qquad (10)$$

where c is a point on the curve C. Since (10) is a solution for both E_1 and E_2, this curve evolution satisfies both of our objectives of having consistent regions with high intra similarity and having dissimilar neighboring regions.

To demonstrate how (10) works, consider the binary image in Fig. 1. Let this image consist of a foreground object I_i and the background I_o. $I_i \cup I_o = R$. The curve C is initialized inside I_i leading to $R_i \subset I_i$. Applying (10) to the points on the curve C, the integral over R_i is equal to zero, whereas the integral over R_o results a positive number expanding the curve towards the object boundaries. If the curve passes the object boundary and reaches the background I_o, (10) becomes negative while shrinking the curve back.
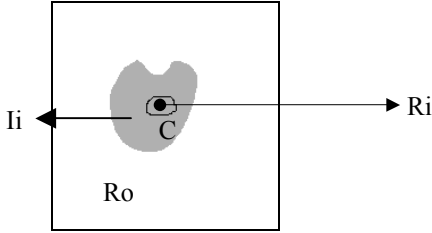

Figure 1. Binary image: A small curve inside the foreground object evolves to capture the object boundaries.

For demonstration purposes, we simplified several things in our formulation of (5) without loss of generality. Equal weightings are assumed for both areas R_i and R_o and in addition, one commonly used criterion, minimizing the curve length, is also ignored. Including these extra conditions, the solution becomes:

$$\frac{\partial C}{\partial t} = \left(\alpha \int\limits_{R_o} w(c,s) ds - \beta \int\limits_{R_i} w(c,s) ds \right) \vec{N} - \gamma \kappa \vec{N} \qquad (11)$$

where κ is the curvature, which smoothes out the boundaries. We choose $\alpha = 1/A_o$ and $\beta = 1/A_i$ in our implementation if (3) is used as the dissimilarity measure. Even though the areas A_i and A_o are not constants but functions of time, it can be shown that this evolution equation still decreases the proposed energy functional, so it is a descent equation. Moreover using the areas as weighting factors has a geometric meaning of normalization. Geometric criteria have been widely utilized in the past for curve evolution [1,2,3] and laid the foundations of GACs.

To apply the proposed curve evolution to vector-valued images, such as color and texture images, there are

two ways of doing it. The first approach, which is the traditional way, is to create N coupled curve evolutions, each applied to one of the dimensions of the N-d vector valued image. In this method, the complexity increases linearly with the number of dimensions. The second approach is to calculate $w(x, y)$ using vector values and not change the PDE. For example, the extended version of (3) is

$$w(x, y) = \sum_N |V_i(x) - V_i(y)| \qquad (12)$$

Since the PDE stays unchanged, the computational complexity also stays the same.

Expanding the segmentation to multiple regions is also possible by recursively applying the bi partitioning to each region.

4. EXPERIMENTAL RESULTS

Fig. 2 shows the flow of the segmentation algorithm. Starting with the input image I, the dissimilarity matrix W and its elements $w(s_i, s_j)$ are calculated using (3) or (4). An initial curve C_0 is propagated under the κ and W based forces until it converges.
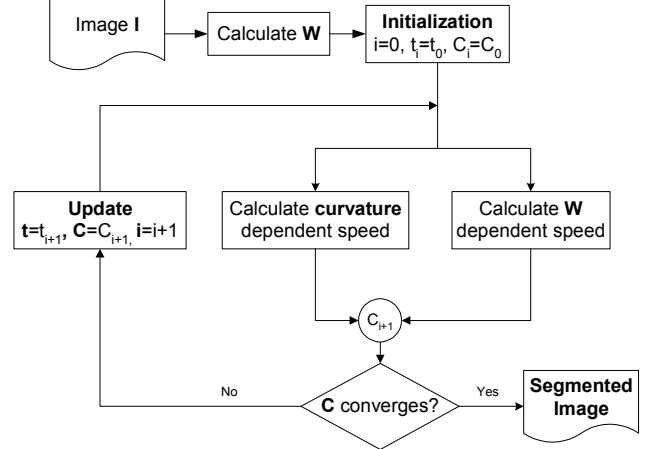

Figure 2. Flow diagram of the segmentation algorithm

The curve can be instantiated manually or automatically placing curves at random or organized places on the image. As demonstrated in Fig. 3, the proposed segmentation method does not depend on the initial location of the curve as much as the edge-based active contours [1]. This is because of the use of all image pixels for the curve movement.

We use the well-known level set method formulation [2,3] to implement the curve evolution in (11). This requires defining a corresponding level set function U that embeds C as its zero level set and the time evolution of U. The level set equation corresponding to (11) is

$$\frac{\partial U}{\partial t} = \left(\alpha \int\limits_{R_o} w(u,s) ds - \beta \int\limits_{R_i} w(u,s) ds - \gamma \nabla \cdot \frac{\nabla U}{|\nabla U|} \right) |\nabla U|$$

Here U is a 3-D function where $U(x, y) = 0$ defines the evolving curve, and u is a point on R. U is generated from the initial curve using the signed distance function

$U(x, y, t = 0) = \pm d$ where d is the distance from (x, y) to C and the sign is chosen positive if (x, y) is outside the contour C and negative if inside the contour.

We have tested the segmentation method on different data sets. The images are intentionally darkened to clarify the evolution of the curve. Segmentation result on an image of blood cells is shown in Fig. 4. w are calculated using intensity values using (3) and an initial curve is evolved to segment the image.

Fig. 3 shows segmentation of a texture image consisting of two regions. A triangle shaped curve is instantiated overlapping both texture regions. w are calculated using (12). The curve evolves and finds the correct boundary.

6. COMPARISON TO OTHER REGION-BASED GACs

Previously used energy functionals focused on approximating the regions with a constant or a smooth function. The curve evolution from Chan et al. [7] uses a piecewise-constant model for the images, where their energy functional serves a similar purpose as the first segmentation criterion we proposed. On the other hand, it doesn't consider the dissimilarity of R_i and R_o. Tsai et al. [5] use $(m_i - m_o)^2$ as the criteria to maximize the separation of the means m_i and m_o of R_i and R_o. This objective is similar to our second segmentation criterion. But this method doesn't consider the homogeneity of the regions itself.

7. REFERENCES

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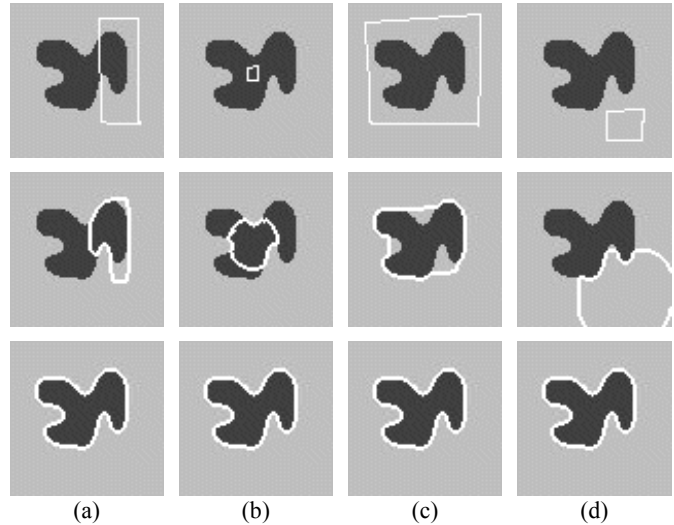[11] Andy Tsai, Appendix A, PhD Thesis, 2000.

(a)　　　(b)　　　(c)　　　(d)

Figure 3. (a-d) Four different types of locations to instantiate the curve and the corresponding curve evolutions converging to the same solution.
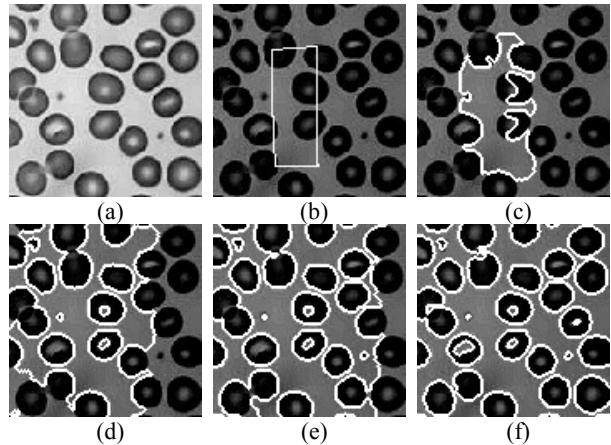

(a)　　　(b)　　　(c)

(d)　　　(e)　　　(f)

Figure 4. (a) Original image (b) Initial curve (c-f) Curve evolution using intensity.
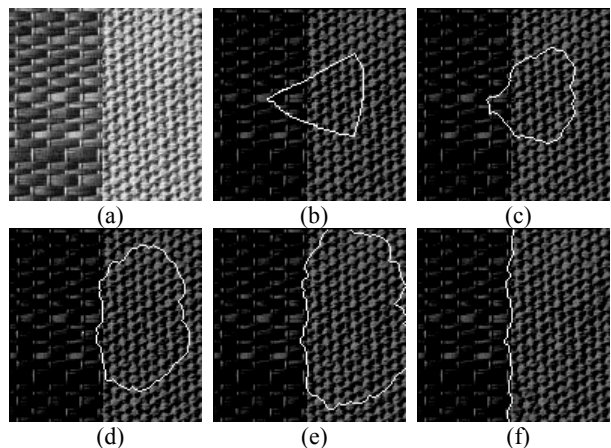

(a)　　　(b)　　　(c)

(d)　　　(e)　　　(f)

Figure 5. (a) Original texture image (b) Initial curve (c-f) Curve evolution using texture features.