OPTIMUM QUANTIZATION ERROR FEEDBACK FILTER FOR WAVELET IMAGE COMPRESION

Zhihai He and S. K. Mitra

Department of Eletrical and Computer Engineering
University of California, Santa Barbara, CA 93106
{zhihai, mitra}@iplab.ece.ucsb.edu

ABSTRACT

In wavelet image compression, all information loss occurs during quantization. Based on the property of bi-orthogonal wavelets, optimal 2-D quantization error feedback filters are designed to reduce the reconstruction error. With very low complexity with regard to computation and implementation the error feedback system improves the PSNR of reconstructed image by about 0.25 dB. In addition, due to its similar structure to the dithered quantizer, it also improves the subjective quality of the reconstructed image by reducing the contouring and ranging effect.

1. INTRODUCTION

In wavelet image compression, all the information loss occurs during the quantization process [1]. Hence, the efficiency of the quantizer plays a key role in image compression algorithm design. In wavelet transform, it is desirable that the associated filters be FIR and linear phase. Hence, bi-orthogonal wavelets are used in wavelet image compression instead of the orthogonal wavelets [2][3]. An efficient quantizer can be designed based on the property of the bi-orthogonal wavelet filter bank.

In digital filter implementation using fixed-point arithmetic, the truncation or round-off error of multiplication operation can be treated as a noise at the output of the filter structure. Error feedback filters can be designed to reduce the noise energy [4]. In this paper, in order to reduce the reconstruction error in wavelet image compression, we have derived and designed optimal 2-D quantization error feedback filters based on the property of the bi-orthogonal wavelet. Moreover, since the proposed error feedback system has a similar structure to the non-substractively dithered quantizer, it also improves the subjective quality of the reconstructed image.

2. STRUCTURE OF 2-D QUANTIZATION ERROR FEEDBACK

Since the transfer function from the quantization error to the reconstruction error is independent with the analysis filter bank, when designing the quantization error feedback filters, we only need to consider the synthesis filter bank. Let the lowpass and highpass synthesis filters be $F_0(z)$ and $F_1(z)$, respectively. For simplicity, let's consider one level of synthesis. The synthesis structure in Fig. 1(a) is equivalent to the structure in Fig. 1(b). Since the four branches in Fig. 1(b) operate separately, without loss of generality, we derive and design the optimal quantization error feedback filter for the 2-D filter $F_{LH}(z_1, z_2) = F_1(z_1) \cdot F_2(z_2)$ in the second branch.

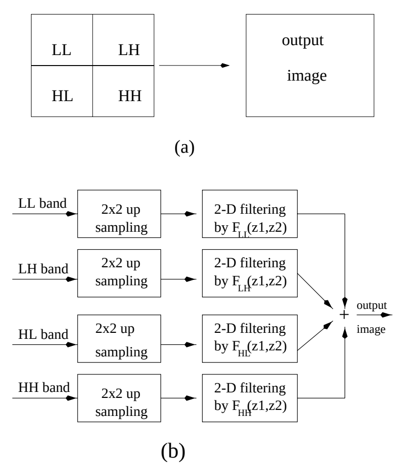

Figure 1: Two equivalent synthesis structures: (a) one level of synthesis; (b) the parallel implementaion by 2-D filtering.

The 2-D error feedback structure used in this paper is shown in Fig. 2(a). e_0 is the quantization error, which is assumed to be white with variance σ^2 when the quantization step size is relatively small. In the

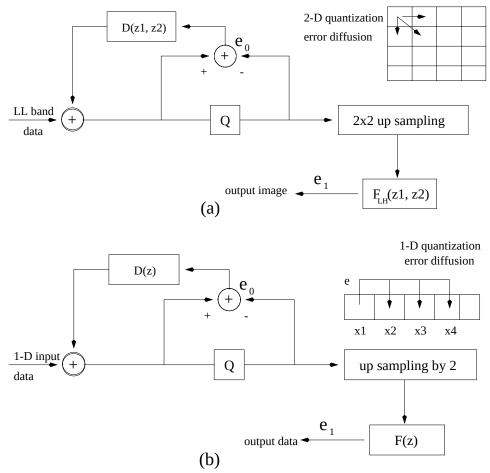

Figure 2: (a) the 2-D quantization error feedback structure. (b) the 1-D quantization error feedback structure.

following section, we will derive the explicit expression of the optimal 2-D filter $D(z_1, z_2)$ in the feedback loop as shown in Fig. 2(a).

3. DERIVATION OF OPTIMAL FEEDBACK FILTER

In order to simplify the derivation procedure of the optimal 2-D error feedback filter $D(z_1, z_2)$, we first compute the optimal 1-D feedback filter for $F(z)$ which is either $F_0(z)$ or $F_1(z)$. The 1-D feedback structure shown in Fig. 2(b) is similar to the 2-D feedback structure shown in Fig. 2(a). The optimal feedback filter $D(z) = \sum_{n=1}^{N} d_n z^{-n}$ should minimize the energy of the reconstruction error e_1. The mean square of e_1 is given by

$$\begin{aligned} MSE &= MSE(d_1, d_2, \cdots d_N) \\ &= \sigma^2 \int_{-\pi}^{\pi} |1 - e^{2jw} D_0(e^{2jw}) F(e^{jw})|^2 dw \quad (1) \end{aligned}$$

To find the minimum point of this function, let

$$\begin{aligned} 0 &= \frac{\partial}{\partial d_k} MSE(d_1, d_2, \cdots d_N) \\ &= 2 \int_{0}^{\pi} \cos 2kw |F(e^{jw})|^2 dw \\ &+ 2 \sum_{n=1}^{N} d_n \int_{0}^{\pi} \cos 2(n-k) |F(e^{jw})|^2 dw. \quad (2) \end{aligned}$$

Written in a matrix form, Eq. (2) becomes

$$\begin{pmatrix} d_1 \\ d_2 \\ \vdots \\ dN \end{pmatrix} = \begin{pmatrix} A_0 & A_1 & \dots & A_{N-1} \\ A_1 & A_0 & \dots & A_{N-2} \\ \vdots & \vdots & \vdots & \vdots \\ A_{N-1} & A_{N-2} & \dots & A_0 \end{pmatrix}^{-1} \begin{pmatrix} B_1 \\ B_2 \\ \vdots \\ B_N \end{pmatrix} \quad (3)$$

where

$$\begin{aligned} A_{|k-n|} &= \int_{0}^{\pi} \cos 2(n-k) |F(e^{jw})|^2 dw, \\ B_k &= (-1) \int_{0}^{\pi} \cos 2kw (F(e^{jw})|^2 dw. \quad (4) \end{aligned}$$

From Appendix (I), we know A is invertible, hence Eq. (3) is valid, and $MSE(d_1, d_2, \cdots d_N)$ has a unique minimum point because from Eq. (1), we have

$$\lim_{||(d_1, d_2, \cdots d_N)|| \to \infty} ||MSE(d_1, d_2, \cdots d_N)|| = \infty \quad (5)$$

Therefore, with the quantization error feedback filter given in Eq. (3), the energy of the reconstruction error is minimized. From Eq. (1), the peak signal-to-noise ratio (PSNR) of the reconstructed image will be increased theoretically by

$$\Delta = 10 \log 10 \frac{\int_{-\pi}^{\pi} |F(e^{jw})|^2 dw}{\int_{-\pi}^{\pi} |F(e^{jw})|^2 |1 - e^{-2jw} D(e^{-2jw})|^2 dw}. \quad (6)$$

Next we derive the optimal feedback filter for the 2-D case as shown in Fig. 2(b). Let the transfer function from the quantization error e_0 to the reconstruction error e_1 be

$$\begin{aligned} T(z_1, z_2) &= D(z_1, z_2) + 1 \\ &= \sum_{k=0}^{N} \sum_{n=0}^{N} T_{kn} z_1^{-k} z_2^{-n}, \quad (7) \end{aligned}$$

where $T_{00} = 1$. The mean square of the reconstruction error is given by

$$\begin{aligned} & MSE = MSE(T) \\ &= \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} |T(e^{j2u}, e^{j2v}) F_{LH}(e^{j2u}, e^{j2v})|^2 du dv. \end{aligned}$$

To find the minimum point, for all l and m such that $l \cdot m \neq 0$, let

$$\begin{aligned} 0 &= \frac{\partial}{\partial T_{lm}} M(T) \\ &= \sum_{k=0}^{N} \sum_{n=0}^{N} T_{kn} A_{kl}^{(0)} A_{nm}^{(1)} \quad (8) \end{aligned}$$

where

$$A_{kl}^{(0)} = \int_{-\pi}^{\pi} \cos 2(k-l)u|F_0(e^{ju})|^2 du,$$
$$A_{nm}^{(0)} = \int_{-\pi}^{\pi} \cos 2(n-m)u|F_1(e^{jv})|^2 dv. \quad (9)$$

Note that in Eq. (3) we already have the explicit solution of the 1-D optimal feedback filter for a given synthesis filter $F_0(z)$ or $F_1(z)$. Let the optimal feedback filter for $F_0(z)$ and $F_1(z)$ be $D_0(z) = \sum_{n=1}^{N} d_n^0 z^{-n}$ and $D_1(z) = \sum_{n=1}^{N} d_n^1 z^{-n}$, respectively. From Eqs.(1) and (2), it is straightforward to show that

$$[T_{kn}] = \begin{bmatrix} 1 \\ d_1^1 \\ d_2^1 \\ \vdots \\ d_N^1 \end{bmatrix} \cdot [\,1 \quad d_1^0 \quad d_2^0 \quad \ldots \quad d_N^0\,] \quad (10)$$

satisfies Eq. (8). From Eq. (7), we know

$$D(z_1, z_2) = [1 + D_1(z_1)][1 + D_0(z_2)] - 1 \quad (11)$$

is the optimal error feedback filter for $F_{LH}(z_1, z_2)$. Following the same procedure, we can also derive the optimal error feedback filters $D_{LL}(z_1, z_2)$, $D_{HL}(z_1, z_2)$ and $D_{HH}(z_1, z_2)$ for synthesis filters $F_{LL}(z_1, z_2)$, $F_{HL}(z_1, z_2)$ and $F_{HH}(z_1, z_2)$, respectively.

4. A DESIGN EXAMPLE

Let's take the Debauchies (5, 3) bi-orthogonal wavelet [3] for example. Its lowpass and highpass filters are

$$F_0(z) = \frac{1}{2\sqrt{2}}(z^{-1} + 2 + z),$$
$$F_1(z) = \frac{1}{4\sqrt{2}}(-z^{-2} - 2z^{-1} + 6 - 2z - z^2)z,$$

respectively. According to Eq. (3), we compute the optimal first-order error feedback filters $D_0(z)$ and $D_1(z)$ for $F_0(z)$ and $F_1(z)$, respectively,

$$D_0(z) = -0.1711z^{-1},$$
$$D_1(z) = 0.1751z^{-1}. \quad (12)$$

From Eqs. (3) and (11), the optimal feedback filters for $F_{LL}(z_1, z_2)$,$F_{LH}(z_1, z_2)$,$F_{HL}(z_1, z_2)$ and $F_{HH}(z_1, z_2)$ are

$$D_{LL}(z_1, z_2) = -0.1711z_1^{-1} - 0.1711z_2^{-1},$$
$$D_{LH}(z_1, z_2) = -0.1711z_1^{-1} + 0.1751z_2^{-1},$$
$$D_{HL}(z_1, z_2) = +0.1751z_1^{-1} - 0.1711z_2^{-1},$$
$$D_{HH}(z_1, z_2) = +0.1751z_1^{-1} + 0.1751z_2^{-1}, \quad (13)$$

respectively.

5. EXPERIMENTAL RESULTS

We apply the error feedback filters in Eq. (13) to the synthesis bank shown in Fig. 2(a) and simulate the synthesis process as shown in Fig. 1(b) on two 512×512 images Lena and Peppers. After one level decomposition of each image, the wavelet coefficients in each subband are quantized at different step sizes with or without quantization error feedback. The quantized wavelet coefficients are encoded by the stack-run image coding algorithm [5]. The rate-PSNR curves for Lena and Peppers are plotted in Fig. 3. It can been seen that with quantization error feedback, the PSNR of output image is improved by about 0.25 dB on average. One unique property of the quantization error feedback is that PSNR will keep increased even when its value is very high.

6. DITHERING EFFECT

At large quantization step sizes, the quantization noise is not independent with the original signal. It is observed that by adding an independent random variable called dither before quantization and subtracting after it, the perceptual quality of the image improves substantially [6]. Wavelet image compression typically introduces two kinds of artifacts into the coded images: contouring and ringing. Contouring refers to the false contours in smooth areas, which is caused by the coarse quantization of the low frequency subbands. The ringing effect refers to the ripples around the edges in the reconstructed image, which is caused by the coarse quantization of high frequency subbands. These artifacts are very visible and subjectively annoying. Dithering can be applied to improve the subject quality. However, it is observed in [6] that dithering can cause an increase in both entropy and distortion.

Our quantization error feedback has a similar structure to the non-substractively dithered quantizer. The only difference is, in non-substractively dithered quantizer a white random noise is added before quantization; but in the error feedback system a deterministic signal is added to the original data before quantization by distributing the quantization error into the neighborhood points as shown in Fig. 2(a). Therefore, we expect that quantization error feedback structure can improve the subjective quality without loss of PSNR. To show this, the quantization error feedback system in Fig. 2(a) is applied to the Lena image at a relative large quantization step size. In Fig. 4, it can be clearly seen that with error feedback the contouring effects on the face and shoulder are reduced. But at the same time the PSNR of output image is increased by 0.27 dB with the optimal quantization error feedback given in Eq. (13).

7. CONCLUDING REMARKS

Based on the property of the bi-orthogonal wavelet, we have derived the explicit expression of the optimal 2-D quantization error feedback filters for the synthesis filter bank. The simulation results show that with quantization error feedback, the PSNR of reconstructed image is improved by about 0.25 dB. In addition, it has been shown that quantization error feedback can also improve perceptual quality due to its dithering effect.

Acknowledgments

This work was supported by a University of California MICRO grant with matching support from Lucent Technologies, National Semiconductor, Tektronix Corp., and Xerox Corp..

8. REFERENCES

[1] M. Antonini, P. Mathieu and I. Daubechies, “Image Coding Using Wavelet Transform,” *IEEE Trans. Image Processing,* Vol. 1, No. 2, April, 1992.

[2] I. Daubechies, *Ten Lectures on Wavelets*, SIAM Philadelphia, PA, 1992.

[3] H. -J. Kim and C. C. Li, “Loossless and lossy image compression using biorthogonal wavelet transforms with multiplierless operations,” *IEEE Trans. Circuits and systems-II: Analog and Ditial Signal Processing*, Vol. 45, No. 8, August 1998.

[4] W. E. Higgins and D. C. Munson Jr., “Optimal and Sub-optimal Error Spectrum Shaping for Cascade Form Digital Filters,” *IEEE Trans. Circuits and System*, CAS-31: 429-437, May 1984.

[5] M. J. Tsai, J. D. Villasenor, and F. Chen, “Stack-run image coding,” *IEEE Trans. Circuits and Systems for Video Technology*, vol. 6, pp. 519-521, Oct. 1996.

[6] D. W. E. Schobben, R. A. Beuker, and W. Oomen, “Dither and Data Compression,” *IEEE Trans. Signal Processing,* vol. 45, pp. 2097 - 2101, August 1997.

APPENDIX (I)

A is a Toeplitz matrix. Suppose there exits $\vec{X} = (x_1, x_2, \ldots, x_N) \in R^N$ such that

$$\vec{X} A \vec{X}^t = 0 \tag{14}$$

Then, from (4), we have

$$\int_0^{\pi} \sum_{k=0}^{N} \sum_{n=0}^{N} x_k x_n \cos 2(n-k)w |F(e^{jw})|^2 dw = 0 \tag{15}$$

Note that

$$|\sum_{k=0}^{N} x_k e^{2jkw}|^2 = \sum_{k=0}^{N} \sum_{n=0}^{N} x_k x_n \cos 2(n-k)w |F(e^{jw})|^2 dw \tag{16}$$

Hence $\vec{X} = 0$, which implies A is invertible. □

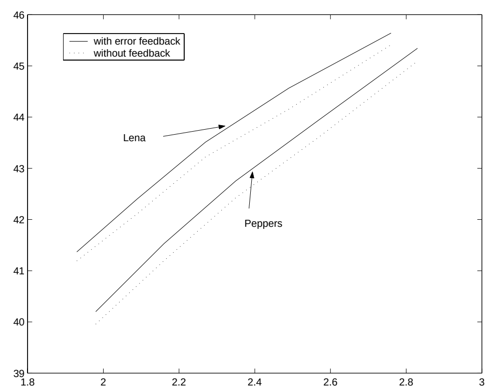

Figure 3: Experimental results for Lena and Peppers.

Figure 4: Contouring effect is reduced by quantization error feedback. Left: reconstructed image without error feedback; Right: reconstructed image with error feedback.