FAST AND ACCURATE RATE PREDICTION AND PICTURE QUALITY CONTROL FOR WAVELET IMAGE CODING

Zhihai He, Tian-Hu Yu, and Sanjit. K. Mitra

Department of Electrical and Computer Engineering
Univeristy of California, Santa Barbara, CA 93106
{zhihai, mitra}@iplab.ece.ucsb.edu

ABSTRACT

By introducing the concepts of *characteristic rate curves* and *rate curve decomposition*, a novel framework for rate-distortion (R-D) analysis and source modeling is developed in this work. With this framework, a fast algorithm is then proposed to accurately estimate the R-D curve of wavelet-based image coders. The proposed algorithm is applied to the SPIHT (Set Partitioning In Hierarchical Trees) [1] and Stact-Run (SR) encoders [2]. Our extensive experimental results show that the relative R-D curve estimation error is less than 5%.

1. INTRODUCTION

The classical R-D formula for a simple quantizer has been developed a long time ago [3, 4]. It is well known that there is a mismatch between the theoretical formula and the actual R-D curve. Meanwhile, it might be very difficult or even impossible to develop a close-form expression for the actual R-D curve [5]. To estimate the R-D curve of a transform encoder, Lin and Ortega [5] first generate eight points on the curve. The whole rate curve is then constructed by cubic interpolation. Ding and Liu [6] model the rate curve with a exponential function. The model parameters are then estimated from the actual coding results. Both methods have very high computational complexity. In addition, they do not provide us with insight into the characteristics of the coding system.

In this paper, by introducing two new concepts of characteristic rate curves and rate curve decomposition, we propose a novel framework for R-D analysis, modeling and estimation. The characteristic rate curves are employed to characterize the input source data accurately and robustly. With rate curve decomposition, the behavior of the coding system is very well approximated. With this framework, an R-D curve estimation algorithm is developed and applied to the SPIHT [1] and the Stack-Run (SR) [2] wavelet-based encoders. Our extensive simulation results show the the relative estimation error is less than 5%.

The paper is organized as follows. In Section 2, we define the characteristic rate curves. Their unique properties are shown in Section 3. In Section 4, an algorithm is developed to estimate these rate curves. We introduce the new concept of rate curve decomposition in Section 5. The R-D curve estimation algorithm and the experimental results are given in Section 6.

2. CHARACTERISTIC RATE CURVES

In this section, we define two characteristic rate curves for the wavelet coefficients. First, the wavelet coefficients are uniformly quantized with stepsize q. Let ρ the percentage of zeros in the quantized wavelet coefficients. Second, we rearrange all the wavelet coefficient into a 1-D array $\mathcal{L}$ in the raster scan order. For each consecutive string of zeros in $\mathcal{L}$, their run length is counted. Let Q_z be the sum of all the sizes of these run-length numbers. Here, the size of a non-zero integer is the number of bits for its binary representation. Let Q_{nz} be the sum of all the sizes of the non-zero wavelet coefficients in $\mathcal{L}$. Obviously, both Q_{nz} and Q_z are functions of the quantization stepsize q, denoted by $Q_{nz}(q)$ and $Q_z(q)$, respectively.

Note that ρ monotonically increases with q. This implies that there is a one-to-one mapping between them. Therefore, Q_{nz} and Q_z are also functions of ρ, denoted by $Q_{nz}(\rho)$ and $Q_z(\rho)$, respectively. For any input image, following the above procedures, we can generate these two curves by varying the quantization stepsize q. These two curves are called the *characteristic rate curves.*

3. PROPERTIES OF THE CHARACTERISTIC RATE CURVES

In this section, we show that the characteristic rate curves have some very interesting properties. To this end, we randomly select 24 sample images with a wide range of R-D characteristics. The sample images are shown in Fig. 1. We plot the two characteristic rate curves for each sample image in Fig. 2. Two observations can be made on these plots. First, the two curves of each sample image have almost the same pattern. This invariance makes it possible for us to estimate the R-D curve accurately and robustly. Second, $Q_{nz}(\rho)$ is almost a straight line passing through [1.0, 0.0]. It should be noted that when the percentage of zeros is 100%, there is no non-zero coefficients. Hence Q_{nz} becomes zero. Let θ be the slope of the straight line.

We have run the above experiment over many other images, the above two observations always hold. In the future work, we shall provide a theoretical explanation for these interesting phenomenon.

4. ESTIMATE THE CHARACTERISTIC RATE CURVES

Based the unique properties of the characteristic rate curves, in this section, we propose a fast algorithm to estimate these two curves. Note that $Q_{nz}(\rho)$ is modeled as a straight line passing through [1.0, 0.0]. We only need to compute one point on the curve to construct the whole rate curve.

A. Estimate $Q_{nz}(\rho)$

Let distribution of the wavelet coefficients be $\mathcal{D}(x)$. It can be approximated by the histogram of the integer parts of the wavelet coefficients, denoted by $\mathcal{H}(n)$. For any given quantization stepsize q_0 and deadzone Δ, the corresponding percentage of zeros is given by

$$\rho_0 = \frac{1}{image\, size} \sum_{n=-\Delta}^{+\Delta} \mathcal{H}(n). \tag{1}$$

According to the definition, we have

$$Q_{nz}(\rho_0) = \sum_{|n|>\Delta} \mathcal{H}(n)[\log_2 \lceil \frac{|n|-\Delta}{q_0} \rceil + 1]. \tag{2}$$

The slope of the rate curve $Q_{nz}(\rho)$ is then given by

$$\theta = \frac{Q_{nz}(\rho_0)}{1-\rho_0}. \tag{3}$$

B. Estimate $Q_z(\rho)$

Surprisingly, we discover that there is a strong linear correlation between θ and the function value of $Q_z(\rho)$. To this end, we watch the values of $Q_z(\rho)$ at ρ_i = 0.6, 0.7, 0.75, 0.80, 0.85 and 0.90 for all the 24 sample images. For each ρ_i, there are 24 pairs of [θ, $Q_z(\rho_i)$] which are plotted in Fig. 3. The correlation coefficients between θ and $Q_z(\rho_i)$ are –0.52, –0.87, –0.87, –0.88, –0.84 and –0.81, respectively. This implies there is a strong linear correlation between $Q_z(\rho_i)$ and the slope θ. Hence, the following linear model

$$Q_z(\rho_i) = A_i\theta + B_i \tag{4}$$

is employed to estimate $Q_z(\rho_i)$. The coefficients A_i and B_i are obtained by statistical regression. The linear correlation models are also shown in Fig. 3.

It might be very difficult to find a theoretical explanation for this interesting linear correlation. But experimentally, it does exists in our extensive simulation over a wide range of images.

5. RATE CURVE DECOMPOSITION

From the above sections, we have observed that in the ρ-domain, the characteristic rate curves have very unique properties. The source data can be characterized by these rate curves much more accurately and robustly in the ρ-domain than in the q-domain. Since these two rate curves characterize the input source data very well, in this section, we will show that the actual R-D curve of the wavelet transform coder can be represented by a linear combination of these two curves.

In functional analysis and digital signal processing, we often study the behavior of a complex function/signal by decomposing it into a series of basis functions/signals with well known properties. Fourier transform and spectrum analysis are good examples [8]. In this work, we apply this methodology to R-D curve estimation and analysis.

Let the actual rate-quantization (R-Q) curve of a wavelet transform coder (for example, the zero-tree [7] or the SPIHT [1] encoder) be $R(q)$. From section II, we know there is a one-to-one mapping between q and ρ. Therefore, we can map $R(q)$ into the ρ-domain and denote it by $R(\rho)$.

In our rate curve decomposition scheme, we approximate $R(\rho)$ by a linear combination of the two characteristic rate curves,

$$\hat{R}(\rho) = \Gamma_1(\rho) \cdot Q_{nz}(\rho) + \Gamma_2(\rho) \cdot Q_z(\rho) + \Gamma_3(\rho). \tag{5}$$

where $\{\Gamma_k(\rho)\}$ are chosen to minimize the L_∞ norm

$$||\hat{R}(\rho) - R(\rho)||_\infty = \max_{0\le\rho\le1} |\hat{R}(\rho) - R(\rho)|. \tag{6}$$

In general, the rate curve is smooth. To simply the computation, we only need to determine the values of $\Gamma_k(\rho)$ at ρ_i = 0.6, 0.7, 0.75, 0.80, 0.85 and 0.90 such that $|\hat{R}(\rho_i) - R(\rho_i)|$ is minimized. This is a least mean square (LMS) problem. For each sample image, we alreay have $Q_{nz}(\rho_i)$, $Q_z(\rho_i)$, and the actual coding bit rate $R(\rho_i)$. $\Gamma_k(\rho_i)$ is then obtained by solving the following linear regression equation

$$R(\rho_i) = \Gamma_1(\rho_i) \cdot Q_{nz}(\rho_i) + \Gamma_2(\rho_i) \cdot Q_z(\rho_i) + \Gamma_3(\rho_i). \quad (7)$$

In our extensive simulation, the probability of the relative approximation error produced by Eq. (5) or (7) being less than 5% is 0.99. This implies it is very accurate to approximate $R(\rho)$ by a linear combination of the characteristic rate curves.

6. ESTIMATION ALGORITHM AND EXPERIMENTAL RESULTS

The characteristics rate curves and rate curve decomposition form a framework for estimating of the R-D curve of wavelet transform coders. For a given wavelet transform encoder, for example, the EZW, the SPIHT or any other one, the decomposition coefficients $\{\Gamma_k(\rho_i)\}$ can be determined as in Section IV. The estimation algorithm is summarized as follows:

- After wavelet transform, compute the approximation histogram $\mathcal{H}(x)$ of the wavelet coefficients. From Eq. (2), $Q_{nz}(\rho)$ is determined. Its slope is then obtained by Eq. (3).
- With the linear correlation model given in Eq. (4), $\{Q_z(\rho_i)\}$ is computed. Based on the decomposition expression in Eq. (7), $R(\rho_i)$ is then estimated.
- The one-to-one mapping between q and ρ can be directly computed from the histogram $\mathcal{H}(x)$. The estimated rates in the ρ-domain can be then mapped into the q-domain. The whole rate curve can be constructed by linear interpolation.

The proposed estimation algorithm is applied to the SPIHT and the Stack-Run (SR) encoder. We arbitrarily choose six test images as shown in Fig. 4. The R-D curves estimation results of the SR and SPIHT coding system are shown in Fig. 5 and Fig. 6, respectively. It can be seen that the estimated curves (dotted ones) are very close to the actual rate-PSNR curves (solid ones). The estimation error is less than 5%.

7. CONCLUDING REMARKS

In this work, we have introduced the concepts of characteristic rate curves and rate curve decomposition, which form the framework for R-D curve estimation and analysis. The proposed algorithm has very low computational complexity and very high estimation accuracy. With the estimated R-D curves, bit rate and picture quality (PSNR) control, bit allocation and R-D optimization can be then performed to improve the encoder performance and transmission efficiency.

Acknowledgments

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8. REFERENCES

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Figure 1: The 24 sample images.

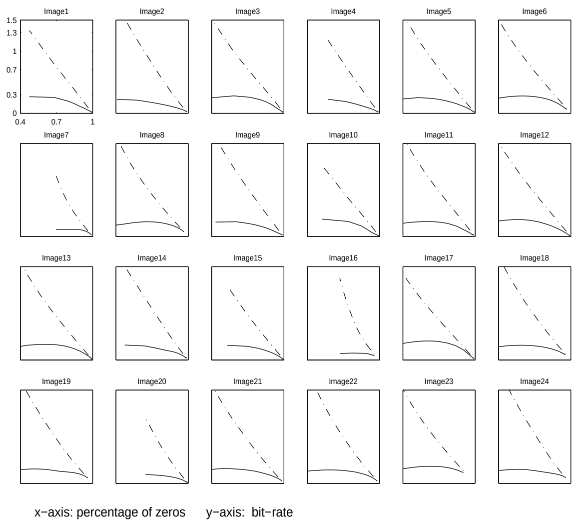

Figure 2: The plots of $Q_{nz}(\rho)$ and $Q_z(\rho)$ for the 24 sample images. The x-axis represents the value of ρ; the dashed curve and the solid curve are function $Q_{nz}(\rho)$ and function $Q_z(\rho)$, respectively. All the subplots have the same axis and labels as the as the first one.

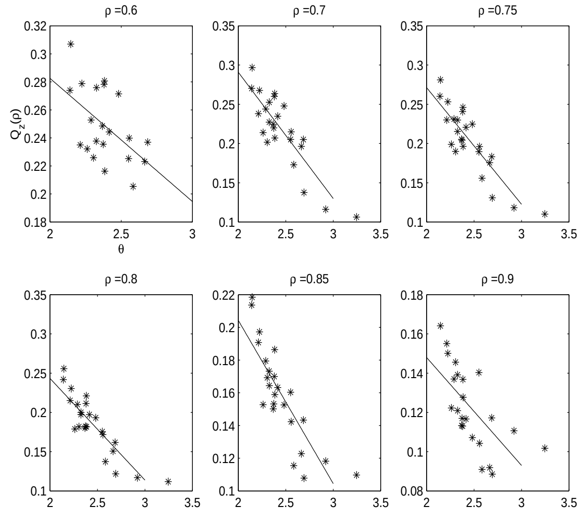

Figure 3: The linear models for the correlation between θ and the values $Q_z(\rho_i)$ at 0.60, 0.70, 0.75, 0.80, 0.85 and 0.90.

Figure 4: The six test images for the evaluation of the proposed algorithm.

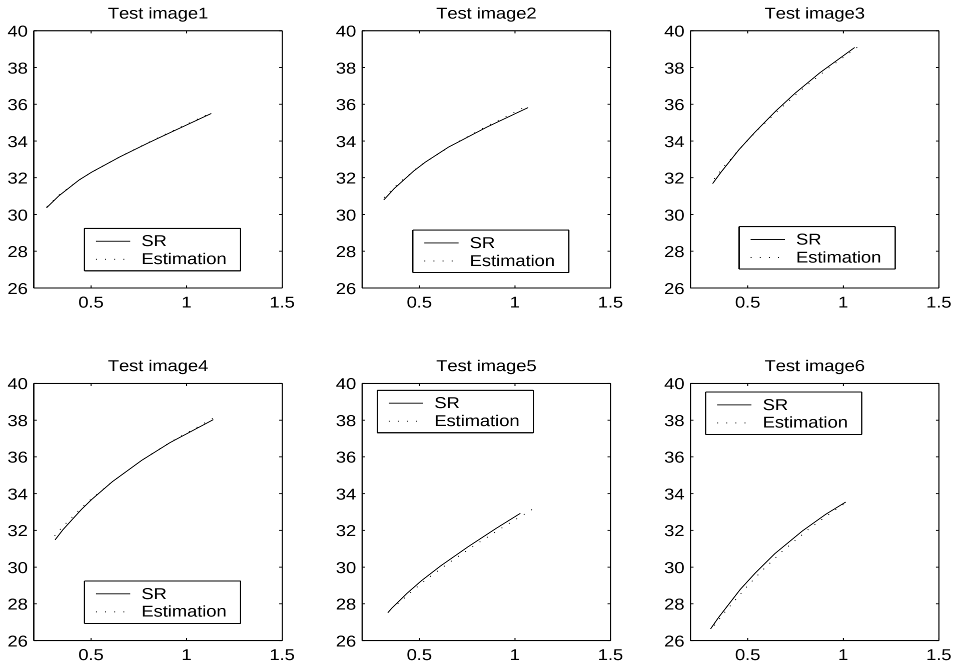

Figure 5: The R-D curve estimation results for the SR coding system.

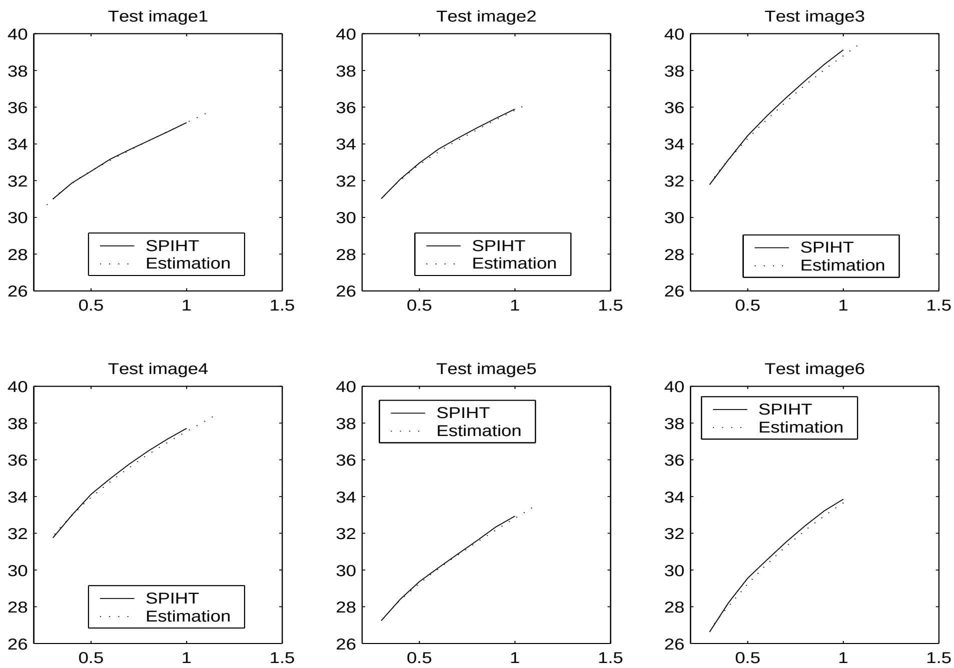

Figure 6: The R-D curve estimation results for the SPIHT coding system.