ECE 278 Final project

Normalized Cuts and Image Segmentation

Baris Sumengen

Jelena Tešic

Vision Research Lab

Electrical and Computer engineering

UCSB

Outline

- Introduction
- Graph Partitioning
- Grouping Algorithm
- Experiments
- Results
- Related Approaches
- Conclusion

Introduction

- Segmentation method based on the paper by Jianbo Shi and Jitendra Malik, CVPR 1997
- Usage of low-level coherence of image attributes for candidate partitions generation
- Set of points is presented as a weighted undirected graph **G=(V, E)**
  - Nodes of the graph – points in the feature space
  - Weight ***W(i,j)*** – “distance measure” for nodes ***i*** and ***j***

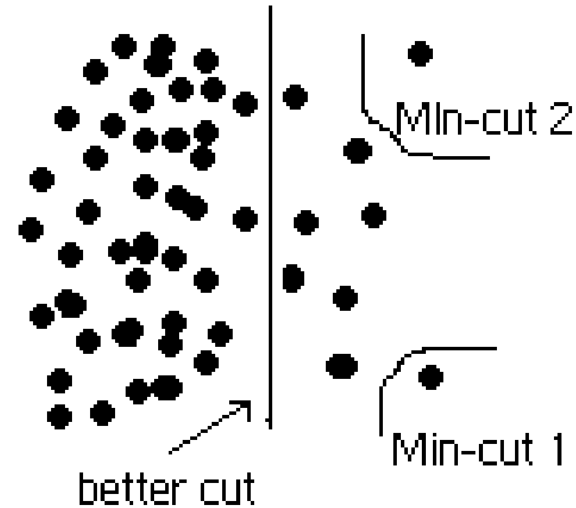
- Graph Partitioning:
  - Precise criterion
  - Efficient partition computation

Graph Partitioning

- Dissimilarity measure between two sets of graph G=(V, E), $A \cup B = V, A \cap B = \emptyset$

$$cut(A,B) = \sum_{u \in A, v \in B} w(u,v)$$

- Minimum cut ⟹ Optimal bi-partitioning
- Clustering method (Wu, Leathy, PAMI 1993) that favors outliers



- Different cost function proposed:

$$Ncut(A,B) = \frac{cut(A,B)}{asso(A,V)} + \frac{cut(A,B)}{asso(B,V)}$$

$$asso(A,V) = \sum_{u \in A, t \in V} w(u,t)$$

Optimal Partitioning

- ***N*** – number of nodes in the graph ***V***
- ***W*** – ***NxN*** symmetrical matrix with elements ***W(i,j)***
- D: $d_i = \sum_j W(i, j) \quad D = diag(d_1, d_2 \ldots, d_N)$
- ***x*** – indication vector: $x_i = \{+1, V(i) \in A \quad -1, V(i) \notin A\}$
- Final result:

$$\min_x Ncut(x) = \min_y \frac{y^T (D - W) y}{y^T D y}$$

with the condition:

$$y_i \in \left\{ 1 \quad \frac{-\sum_{x_i > 0} d_i}{\sum_{x_i < 0} d_i} \right\}, \quad y^T \begin{bmatrix} d_1 \\ \vdots \\ d_N \end{bmatrix} = 0$$

Optimal Partitioning- con't

- Solution comes from eigenvalue system: $(D-W)y = l\,Dy$
- Second constraint is satisfied for any eigenvector.

$$D^{-1/2}(D-W)D^{-1/2}z = l\,z \qquad z = D^{1/2}y$$

Symmetric, semi-positive definite – Laplacian matrix

- Using Rayleigh quotient and orthonormality of eigenvectors: Real valued solution to the Normalized Cut problem is the second smallest eigenvector of the generalized eigensystem
- Graph is partitioned in two pieces using the second smallest eigenvalue
- Ideal case: the signs of vector elements tells us how to split

The Grouping Algorithm

- Use spatial proximity term and feature similarity terms to calculate weighted coefficients ***W(i,j)***
- Summarize information in ***W*** and ***D***
- Find an approximate solution for the second smallest eigenvalue of the system $(D-W)y = l Dy$
- Bipartition the graph by finding the best splitting point
- Check the stability of the cut and decide on the current partition status
- Stop the recursion if Ncut exceeds certain limit

Experiments

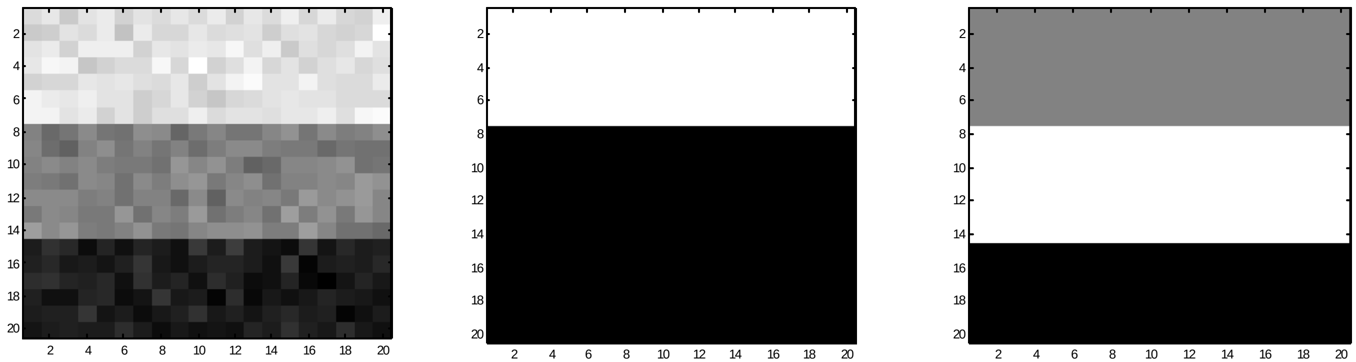
- Different parameter values and feature desriptors

$$W(i,j) = e^{\frac{-\left\|F_i - F_j\right\|_2}{s_I}} * e^{\frac{-\left\|X_i - X_j\right\|_2}{s_X}}, \left\|X_i - X_j\right\|_2 < r$$

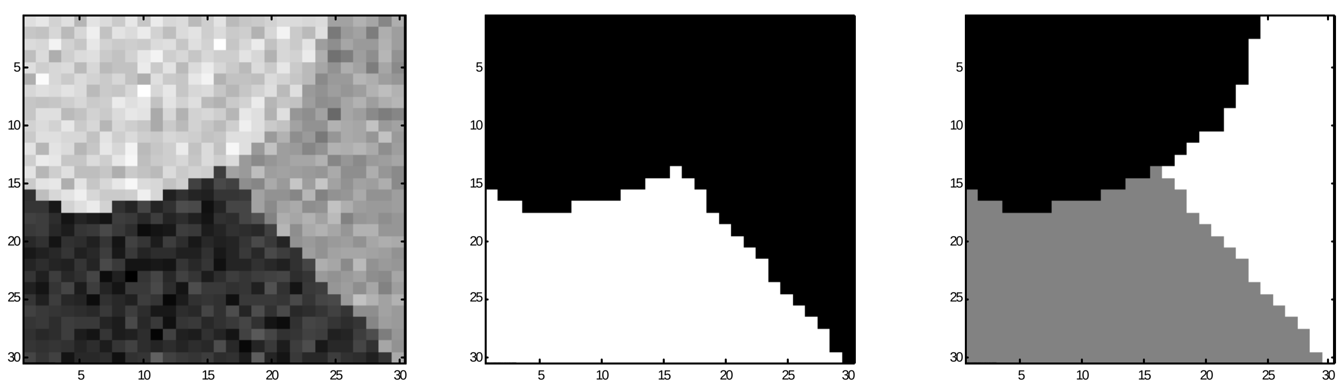
$$F(i) = \begin{cases} I(i), & \text{intensity} \\ [v, v \cdot s \cdot \sin(h), v \cdot s \cdot \cos(h)](i), & \text{color} \\ [\left|I * f_1\right|, \ldots, \left|I * f_n\right|](i), & \text{texture} \end{cases}$$

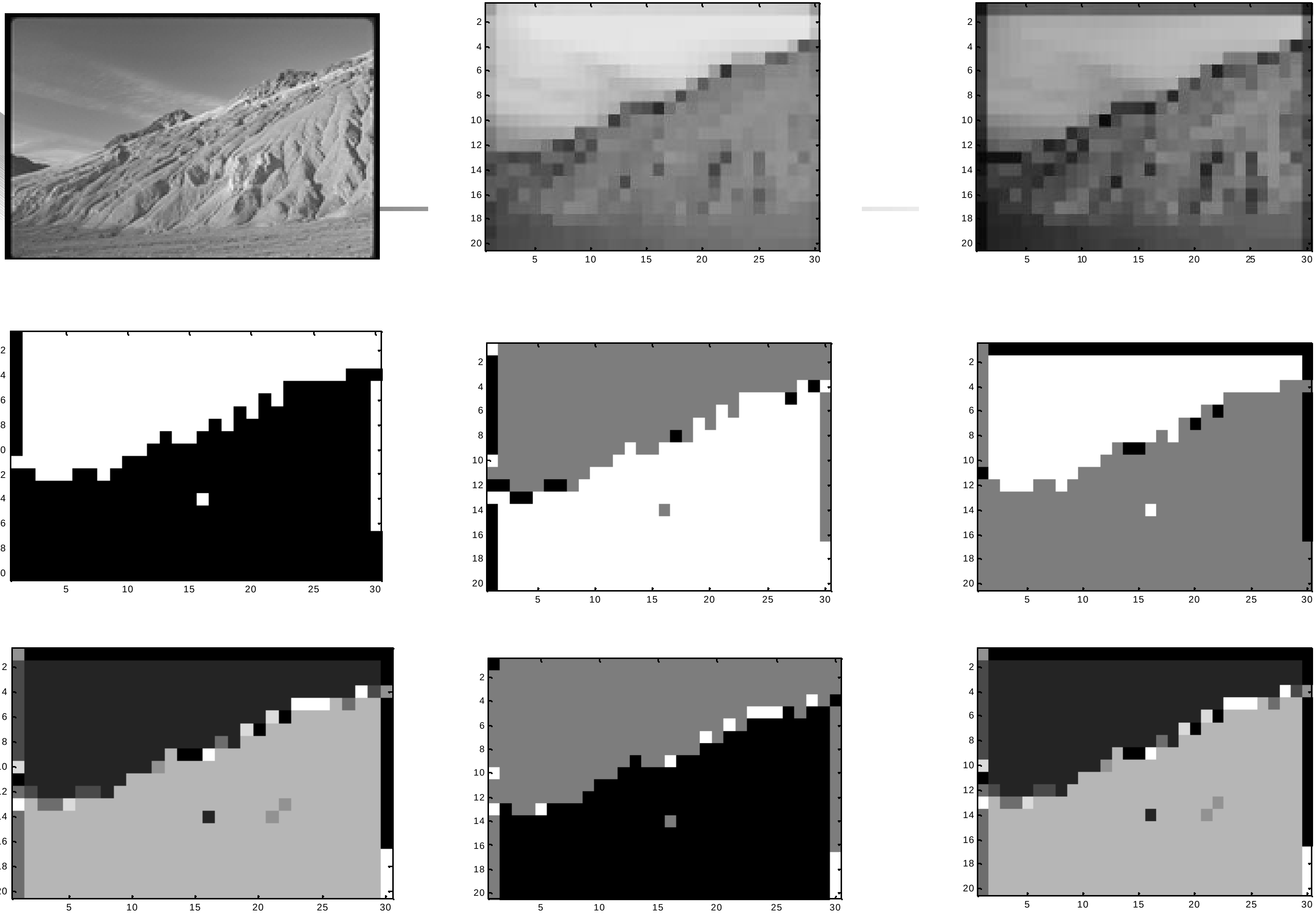
Results

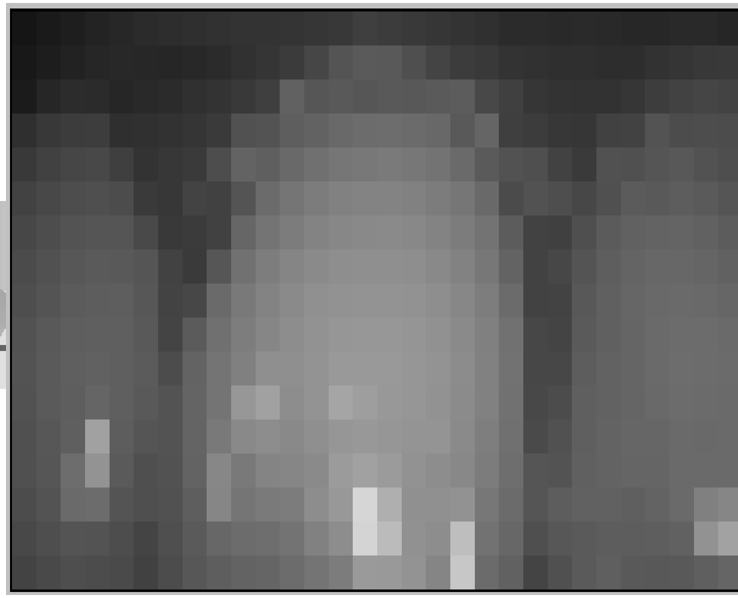
$$s = 0.1, r = 3.0, s_I = 0.02, s_X = 3.0$$

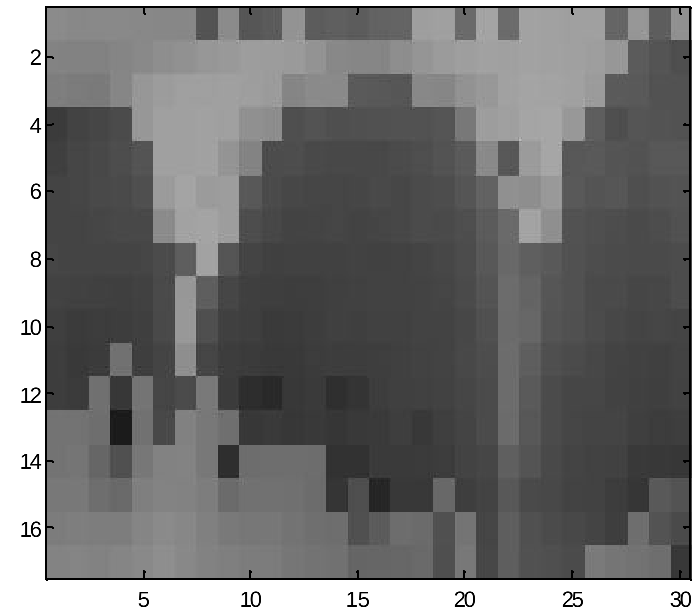


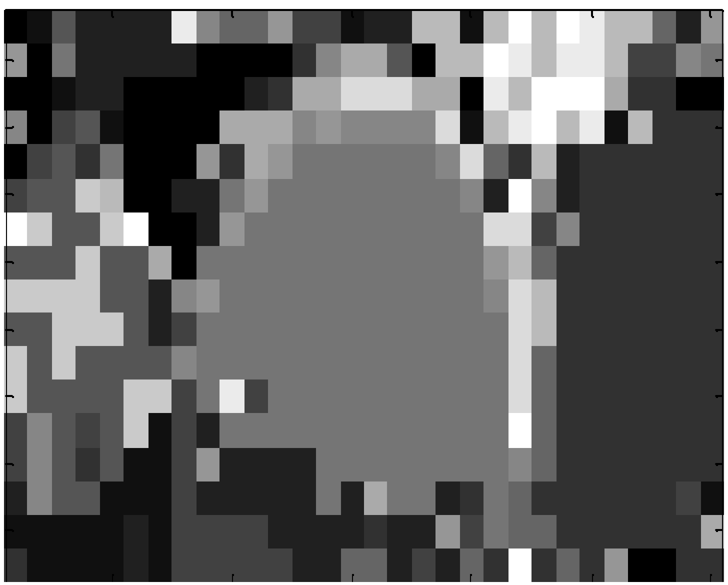
$$s = 0.1, r = 5.0, s_I = 0.01, s_X = 4.0$$

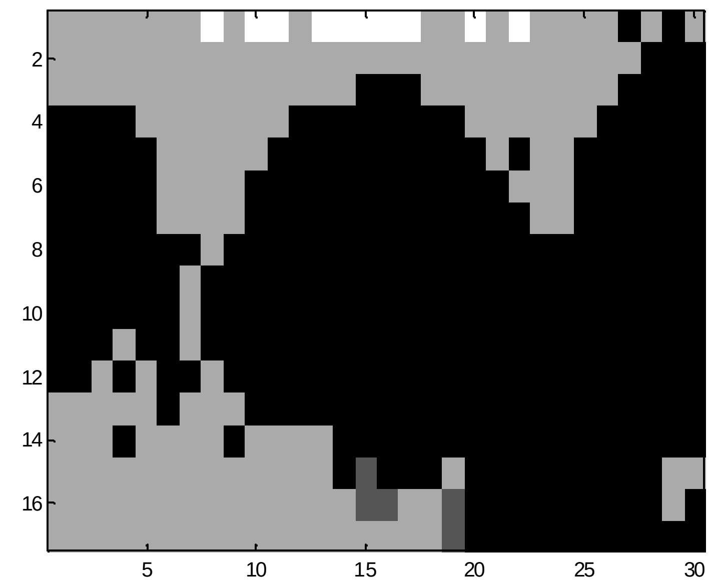







2
4
6
8
10
12
14
16
5 10 15 20 25 30




2
4
6
8
10
12
14
16
5 10 15 20 25 30

Conclusion

- Computational Complexity
- Choice of Parameters
- Stability
- Heuristic methods
- Application