ADJUSTABLE QUADRATIC FILTERS FOR IMAGE ENHANCEMENT

Reinhard Bernstein

Telecommunications Institute
University Erlangen-Nuremberg, Cauerstr. 7
D-91058 Erlangen, Germany

Michael Moore and Sanjit Mitra

Dept. of Electrical and Computer Engineering
University of California, Santa Barbara
Santa Barbara, CA 93106-9560

ABSTRACT

We propose a new class of mapping-based nonlinear quadratic Volterra filters for image enhancement. The input signal is mapped prior to filtering by a memoryless one-to-one nonlinear mapping function. The mapping function can be described by any arbitrary function and implemented by a simple look-up table. If necessary, an inverse mapping function is applied after filtering. Using this general approach, the quadratic Volterra filter output can be adjusted to a desired response. Some results are presented where a highpass quadratic Volterra filter is used to enhance an image by unsharp masking.

1. INTRODUCTION

In its general form, a 1-D quadratic digital Volterra filter is given by the 2-D convolution of the 1-D sample products $x(n_1) \cdot x(n_2)$ with a 2-D kernel $h_2(n_1, n_2)$. Specifically,

$$y(n) = \sum_{k_1=-\infty}^{\infty} \sum_{k_2=-\infty}^{\infty} h_2(k_1, k_2) \cdot \; x(n-k_1)x(n-k_2). \tag{1}$$

For example, Teager's algorithm [1], which estimates a measure of the signal energy, is a 1-D quadratric Volterra filter defined by

$$y(n) = x^2(n) - x(n-1) \cdot x(n+1). \tag{2}$$

As discussed in [2], the output from Teager's algorithm is approximately equal to

$$y(n) \approx \mu[(x(n) - x(n-1)) + (x(n) - x(n+1))] \tag{3}$$

where $\mu = (x(n-1) + x(n) + x(n+1))/3$. Thus the output of a Teager filter is approximately equal to a highpass filter response weighted by the local mean.

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The 1-D Teager filter can be extended to create 2-D Teager-like filters [2], [3]. The 2-D Teager filter, like the 1-D Teager filter, is roughly equivalent to a local mean weighted 2-D Laplacian filter and requires fewer computations to implement. When the filter is applied to an image, the response is stronger in regions of high average intensity than in regions of low average intensity. The filter output can then be used to enhance an image by unsharp masking. Because of the mean weighting effect of the Teager filter, high frequency edges, such as those due to impulse noise, are not enhanced as much in dark regions. This characteristic is a good match for the human visual system [3].

In other applications, it may be desireable to adjust the degree to which the highpass filter output depends on the average local intensity. In some cases, one may even want to enhance high frequency information in darker regions more than in brighter regions. This paper explores a relatively simple method for adjusting the dependence on the local average intensity by premapping the input signal, applying a 2-D Teager filter to the resulting signal, and then applying an inverse mapping to its output, if required.

2. MAPPING-BASED ENHANCEMENT BY QUADRATIC FILTERING

The sensitivity of the 1-D Teager filter to the local mean can be adjusted by raising each term in eq. (2) to a fractional power. Specifically,

$$y(n) = x^{\frac{2}{m}}(n) - [x(n-1) \cdot x(n+1)]^{\frac{1}{m}}. \tag{4}$$

Like the original function, this equation can be approximated as

$$y(n) \approx \frac{\mu^{\frac{2}{m}-1}}{m}[(x(n)-x(n-1))+(x(n)-x(n+1))] \tag{5}$$

where, as before, μ is the local mean. The weighting of the local mean can be adjusted by changing the parameter m. For $m < 2$, the highpass output is weighted more heavily in high intensity regions. When $m > 2$, the highpass output is weighted more heavily in low intensity regions. For $m = 2$, the filter output does not depend on the local mean and approximates the output of a Laplacian filter.

Note that eq. (4) can be rewritten as

$$y(n) = (x^{\frac{1}{m}})^2(n) - [x(n-1)^{\frac{1}{m}} \cdot x(n+1)^{\frac{1}{m}}]. \quad (6)$$

Therefore, the filter can be implemented by taking the mth root of each incoming pixel value and then filtered using a preexisting Teager filter to produce the output.

Figure 1 describes our proposed implementation of the adjustable Volterra filter. Without any loss of generality, we normalize x to a range of [0,1] where a one corresponds to the maximum possible greylevel. The input mapping function maps each pixel value to its mth root or, more generally, to any desired function of the pixel values. In some cases, an output mapping stage may be required. Denormalization remaps the filter output signal y to the original dynamic range of the input signal.

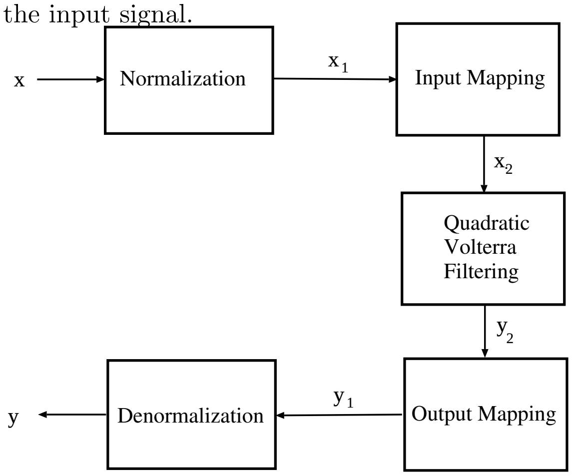

Figure 1: Mapping based Volterra filtering

Note that the normalization stages are not strictly necessary and could be combined with the mapping stages. The normalization steps simply ensure that the input and output ranges of the Teager filter are the same. In general, the range of the Teager filter output will be equal to the square of the range of the input signal. When scaled and combined with the original image, the output of this filter can be used to perform unsharp masking.

Figure 2 shows plots of some sample mapping functions. The mapping functions are:

dashdot line:

$$f_{map1}(x) = x^{\frac{1}{2}} = \sqrt{x} \quad (7)$$

dashed line:

$$f_{map2}(x) = x^2 \quad (8)$$

solid line:

$$f_{map3}(x) = x \quad (9)$$

circle marked line:

$$f_{map4}(x) = \begin{cases} 1 - \sqrt{\frac{1-x}{2}} & 0.5 < x \leq 1 \\ \sqrt{\frac{x}{2}} & 0 \leq x \leq 0.5 \end{cases} \quad (10)$$

cross marked line:

$$f_{map5}(x) = \begin{cases} 1 - 2(1-x)^2 & 0.5 < x \leq 1 \\ 2x^2 & 0 \leq x \leq 0.5. \end{cases} \quad (11)$$

Mapping functions (7) through (9) implement mth root mappings. Functions (10) and (11) combine features of some of the other functions to adjust the intensity band emphasized by the filter. Function (10) enhances both very light and very dark areas. Function (11) tends to enhance middle intensity regions more than the intensity extremes. Output mapping is not required for any of these functions.

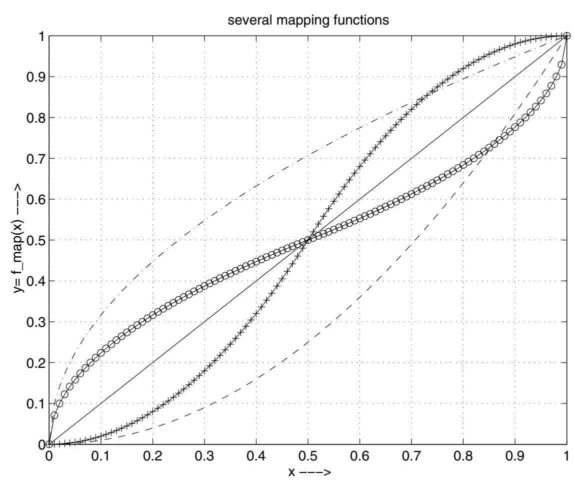

Figure 2: Mapping functions (7)-(11)

The darker portions of an image become increasingly enhanced as m increases above 2. However, the overall response of the filter decreases as m increases due to the $\frac{1}{m}$ term in eq. (5). Therefore, this approach becomes limited. Another approach is to use an inverse mapping prior to the Teager filter operation with a value of m less than 2. The output of the Teager filter with inverted inputs is

$$Teager(1-x) = Teager(x) - Laplacian(x), \quad (12)$$

where,

$$Laplacian(x) = 2x[n] - x[n-1] - x[n+1]. \qquad (13)$$

This relationship is exact. If we substitute the Teager filter approximation from eq. (3) into eq. (12),

$$Teager(1-x) \approx (\mu - 1) \cdot Laplacian(x). \qquad (14)$$

If we use a postmapping of $y_1 = -y_2$ then the overall response is approximately equal to a highpass filter response weighted by the inverse of the local mean. Note, that this is not the same as simply reflecting the Teager response around the midrange value of 0.5. However, the response does decrease as the average local intensity increases, as desired.

3. SOME EXAMPLES

Figure 3 provides some 1-D examples of responses to the systems that have been discussed. Plot (a) is the input signal and contains three impulses at various levels and two step changes. Plots (b), (c), and (d) show the output for some mth root mappings. The response increases with average intensity for $m < 2$, is relatively constant for $m = 2$, and decreases with average intensity for $m > 2$. Plot (e) shows the response for f_{map5}. This function enhances the middle intensity impulse more than the high or low average intensity impulses. Finally, plot (f) shows the output of the inverse mapping described at the end of the last section. The response decreases with average intensity. Note that the response is not the mirror image of the response for $m = 1$, which is the normal Teager filter response.

Figure 4 demonstrates the difference in results that can be obtained by changing the mapping function. The original image is an x-ray of a human jaw. The Teager-like filter used to highpass filter these images is described by eq. (53) in [3]. The images were enhanced using unsharp masking.

4. REFERENCES

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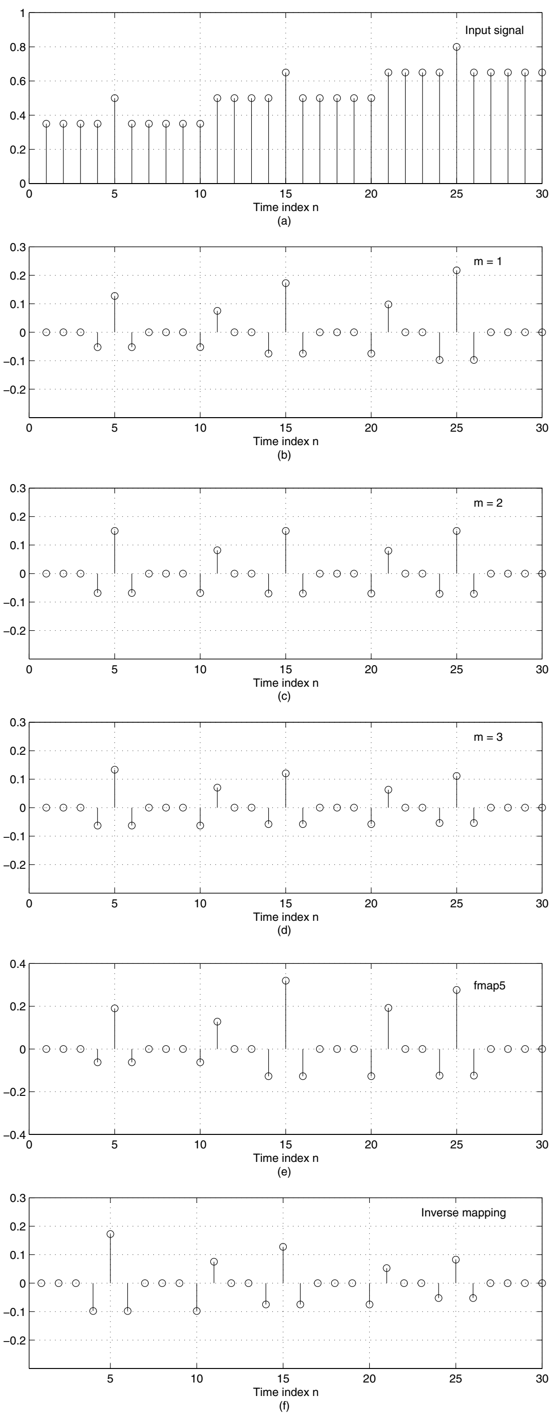

Figure 3: Generalized Teager filter test input response.

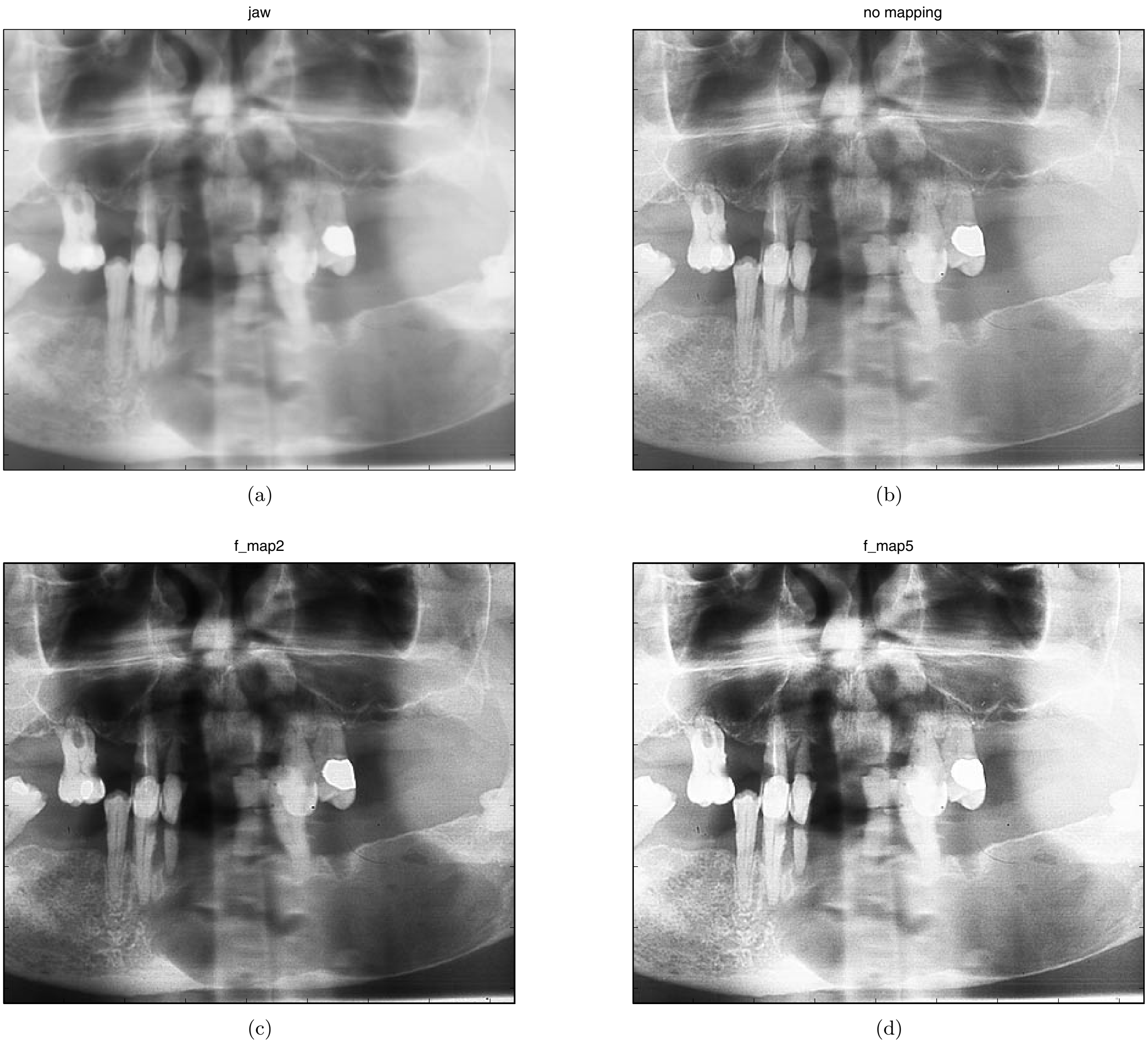

Figure 4: Enhancement examples using the generalized Teager algorithm. Image (a) is the original image of a human jaw. The other images were enhanced using a 2-D Teager filter with (b) no premapping, (c) mapping function f_{map2}, and (d) mapping function f_{map5}.