

# NOVEL CODING SCHEME FOR WAVELET IMAGE COMPRESSION

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## ABSTRACT

In this paper, we propose a very simple image compression scheme. The proposed coding scheme employs the multi-level dyadic wavelet decomposition, linear quantization with a proper dead zone, and then it converts the quantized wavelet coefficients into three 1-D sequences for adaptive arithmetic coding. First, some of the clustered zeros in the multi-level dyadic wavelet decomposition are represented with the quadtree code, and then the remaining zeros and non-zero integers are arranged as a 1-D sequence by raster scanning. Next, the resulting integer sequence is decomposed into two 1-D sequences with small alphabet. Despite the simplicity of the proposed coding scheme, the rate-distortion performance of the proposed image compression algorithm is competitive with the best image coders in the literature.

## 1. INTRODUCTION

In many different fields such as digital cameras or computer tomography, digital images are replacing conventional analog images. The volume of data required to describe such images leads to greatly slower transmission and makes storage prohibitively costly. Therefore, the information contained in the data has to be compressed by extracting the principal elements, which are then encoded, and consequently, the quality of the reconstructed image with the principal elements is reduced. A fundamental goal of digital image compression is to reduce bit rates for transmission or storage while maintaining an acceptable quality of reconstructed images. Meanwhile, the complexity of the compression algorithm has to be as low as possible so as to be implemented with reasonable cost.

To this end, many image compression techniques have been developed, such as transform image coding, predictive image coding and vector quantization. Among these techniques, transform image coding is most efficient, particularly at low bit rates. There are

basically three steps in transform image coding: transformation, quantization, and entropy coding. Because wavelet transform coefficients are well localized in both space and frequency domains, wavelet image coding has proven to be an efficient transform image compression technique. Recently, some advanced wavelet transform image coding algorithms have been developed, of which the popular ones are the embedded zerotree wavelet transform coding algorithm (EZW) [1], the coding algorithm based on set partitioning in hierarchical trees (SPIHT) [2], and the stack-run coding method (SR) [3]. These three wavelet image coding algorithms adopt similar schemes in all the three steps: multi-level dyadic wavelet transform, linear quantization with a proper dead-zone, and adaptive arithmetic entropy coding. The main difference among these three algorithms is in the data structure representing the quantized wavelet coefficients for adaptive arithmetic entropy coding.

In addition, some other wavelet transform image coding algorithms have been introduced to improve the performance of the popular ones based on different motivations. Wu and Chen [4] developed some simple context modeling techniques to squeeze out more statistical redundancy in the wavelet coefficients of EZW-type image coders so as to improve the coding efficiency. Joshi et al. [5] applied the classification technique to subband image coding in order to exploit the nonstationary nature of the image subbands. Based on an optimal classification of subbands, an optimal rate allocation can be reached. The choice of quantization method is a crucial issue to improve performance in wavelet image compression. Xiong et al. [6] optimally select the spatial zero-tree regions and optimally choose the ‘uniform’ scalar quantizer for remaining coefficients to improve upon EZW. Zhang and Fisher [7] modified the EZW and SR algorithms by replacing scalar quantization with trellis-coded quantization (TCQ). Besides, Tran and Nguyen [8] replace the dyadic wavelet transform by M-channel uniform band maximally decimated linear phase perfect reconstruction filter banks in or-

der to obtain finer frequency spectrum partitioning and higher energy compaction.

Though these modified wavelet image compression algorithms may be more efficient than those of the popular ones, they usually suffer from high computational complexity. In general, with a particular coding technique, efficient coding algorithms are generally complicated and simple coding algorithms are usually inefficient. For example, the SR coding scheme is simpler than the SPIHT coding scheme, but the efficiency of the SR algorithm is less than that of the SPIHT algorithm. Consequently, a tough job in image coding is to develop a coding algorithm that is not only efficient in coding but also is simple in implementation. There usually are two methods to design simple and efficient coding algorithms. One is to modify a simple coding algorithm by keeping its simplicity while improving its efficiency; the other is to modify an efficient coding algorithm by keeping its efficiency while reducing its complexity.

Recently, we introduced an efficient wavelet image compression scheme, in which we use 1-D variable length block coding to exploit the clustered zeros of the 1-D quantized wavelet coefficient sequence [9]. This compression scheme possesses the simplicity of the SR coding scheme while reaching the efficiency of the SPIHT algorithm. However, like the SR coding scheme, it is basically a 1-D sequence processor so that it could not exploit the 2-D nature of wavelet zeros efficiently. The quadtree decomposition method is an efficient technique for describing 2-D regions in image processing, and it has been used in different image coding algorithms [10] - [12]. In this paper, we take the quadtree code as a 2-D variable length square block coding method to exploit the 2-D nature of wavelet zeros. The proposed coding scheme employs the multi-level dyadic wavelet decomposition, linear quantization with a proper dead zone, and then it converts the quantized wavelet coefficients into three 1-D sequences for adaptive arithmetic coding. First, some of the clustered zeros in the multi-level dyadic wavelet decomposition are represented with the quadtree code, and then the remaining zeros and non-zero integers are arranged as a 1-D sequence by raster scanning. Next, the resulting integer sequence is decomposed into two 1-D sequences with small alphabets.

## 2. BASICS OF WAVELET IMAGE COMPRESSION

The principle behind transform image coding is based on two factors: the correlation among the transform coefficients is reduced so that redundant information

does not have to be coded repeatedly, and because of the energy compaction property it is possible to code only a fraction of the transform coefficients without producing serious distortion. In transform image coding, an image is first transformed to a domain significantly different from the image intensity domain, then the transform coefficients are quantized with a finite number of values, and finally these quantized transform coefficients with many clustered zeros are coded with entropy coding. Different transforms employ different transform structures, for example, in DCT transform coding, an image is divided into small blocks to be transformed, but in wavelet transform coding, the whole image is transformed by a dyadic decomposition. Since the discrete wavelet transform is well localized in both the space and frequency domains, it gives good compression results [1]- [8]. Therefore, it has received much attention. Both vector quantization and scalar quantization have been used in transform image coding. However, linear scalar quantization with a proper deadzone is very popular in wavelet transform coding. As to entropy coding, adaptive arithmetic coding has been proven to be more efficient than Huffman coding.

The quantized transform coefficients are usually represented by an integer data set. In general, the range of the set is very large, and because of the energy compaction property, the set contains many clustered zeros. As we know, in adaptive arithmetic coding, a large alphabet of the input data not only increases the computational complexity and memory usage, but also makes it very difficult to estimate the conditional probabilities by frequency counts within a single image, which is called the context dilution problem. Thus, it can not be efficient to code the set directly with adaptive arithmetic coding, and consequently, a tough job in transform coding is to find an efficient data structure to represent the 2-D nature of quantized transform coefficients for entropy coding. To represent clustered zeros, JPEG uses a symbol EOB to represent a zero run. EZW and SPIHT use a symbol for the zero tree root to represent a special structure of clustered zeros within subbands, and SR uses binary code to represent zero runs. To solve the dilution problem, we have to find a way to convert a data set with a large alphabet into data sets with small alphabets. To this end, JPEG employs a set of special symbols to jointly represent a non-zero integer and a related zero run. EZW and SPIHT use a bit-plane coding like scheme to convert the integers into two 1-D sequences: dominant pass and subordinate pass. The SR coding uses binary code to represent the non-zero integers.

### 3. PROPOSED WAVELET IMAGE COMPRESSION ALGORITHM

Our new wavelet coding scheme is similar to EZW, SPIHT, and SR in the three steps: multi-level dyadic wavelet transform, linear quantization with a proper dead-zone, and adaptive arithmetic entropy coding. After a multi-level dyadic subband decomposition and quantization, we make three observations: (1) there are many zeros, particularly at low bit rates with large step sizes, (2) most of the zeros are clustered into 2-D groups of different sizes, and (3) the higher the frequency of the related subband is, the larger the size is.

For a binary image (black and white), quadtree code has proven to be a powerful technique for describing 2-D regions. With the quadtree decomposition method, for a given minimum size of the square block, a binary image is divided into white square blocks of different sizes, black square blocks of different sizes, and white-and-black blocks of the minimum size. This decomposition is represented by the 1-D quadtree code obtained in the decomposition procedure. In order to exploit such a 2-D nature of quantized wavelet zeros for adaptive arithmetic entropy coding, we use quadtree code to represent the zero square blocks of different sizes. In the first stage of the decomposition procedure, the quantized dyadic wavelet decomposition image is considered as a binary image with two symbols: zero and non-zero. In the decomposition, each square block is examined if all the coefficients of the block is zero (white) or non-zero (black). Otherwise this block (white-and-black) is divided into four quadrants for further examination until a given minimum block size is reached. The resulting quadtree code is usually represented by a unique string of symbols “b” (black), “w” (white), and “g” (white-and-black). To represent the resulting quadtree code more efficiently for adaptive arithmetic coding, we merge two symbols “b” and “g” into symbol “1” and use symbol “0” to represent “w”. If the symbol “1” represents a black block with a size larger than the minimum size, it is followed by four 0s so as to avoid an ambiguity for representing both the black block and the gray block.

In the second stage of the decomposition procedure, the coefficients inside the black blocks and gray blocks are raster scanned into an 1-D integer sequence in the quadtree decomposition order. Since the resulting integer sequence usually does not contain many clustered zeros, the stack-run code [3] is not suitable to efficiently represent it. To solve the dilution problem, we develop a data decomposition method to convert the generated integer sequence into two sequences with small alpha-

bets. Let  $i$ ,  $p(i)$  and  $(-N, N)$  be an integer to be entropy-coded in a data source  $S$ , its probability and dynamic range. The entropy of  $S$ ,  $H(S)$ , is given by

$$H(S) = - \sum_{i=-N}^N p(i) \log_2 p(i). \quad (1)$$

In order to reduce the alphabet, the data source  $S$  is divided into sub-data sources  $g(k)$ ,  $k = 0, \dots, K$ , and  $N \leq 2^K$ .  $g(k)$ ,  $k \geq 0$  contains the integers in the range  $[2^{k-1}, 2^k - 1]$  and  $[-2^k + 1, -2^{k-1}]$ . Note that  $g(0)$  contains only one integer, 0.

Let  $H(g(k))$  be the contribution of the integers in the group  $g(k)$  and  $p(g(k))$  be the probability of the symbol of the group  $g(k)$ . Thus, we have

$$H(g(k)) = - \sum_{i \in g(k)} p(i) \log_2 p(i), \quad (2)$$

and

$$p(g(k)) = \sum_{i \in g(k)} p(i). \quad (3)$$

For an integer  $i$  belonging to  $g(k)$ , we use  $p_k(i)$  to denote  $p(i)/p(g(k))$ , and  $H_k$  is given by

$$H_k = -p(g(k)) \times \sum_{i \in g(k)} p_k(i) \log_2 p_k(i), \quad (4)$$

and thus,  $H(g(k))$  can be rewritten as

$$H(g(k)) = -p(g(k)) \log_2 p(g(k)) + H_k. \quad (5)$$

Consequently,  $H(S)$  can be rewritten as

$$H(S) = \sum_{k=0}^K (-p(g(k)) \log_2 p(g(k)) + H_k), \quad (6)$$

where  $H_0 = 0$ . With the above data decomposition method, the entropy-coding of the integer sequence can be implemented by entropy-coding two sequences: group symbol sequence and sequence corresponding to  $H_k$ .

The data decomposition method can be applied to the generated integer sequence as follows. In binary representation of non-zero integers, there are three parts: sign bit, most significant bit (MSB), and remaining part called less significant bits (LSB). Because MSB is always “1” it does not contain any information itself, but the MSB bit position contains information of the non-zero integer size. We define the MSB bit position as length and the combination of the sign bit and LSB as residue. Specially, the length of the integer zero is assigned to be zero, and it has no residue. For example, the length of one (or minus one) is one,

and its residue contains only the sign bit. Similarly, the length of three is two, and its residue is "01". Based on the above definitions, an integer sequence can be converted into two sequences, to be called an L-sequence (group symbol sequence) and an R-sequence (sequence corresponding to  $H_k$ ), where the  $i$ -th element of the L-sequence,  $L(i)$  = the length of  $I(i)$ , the  $i$ -th element of the integer sequence, and the  $i$ -th element of the R-sequence,  $R(i)$  = the residue of  $I(i)$ .  $L(i)$  is a non-negative integers, whereas  $R(i)$  is either a "0" or a "1".

Thus, with the quadtree decomposition method and the data decomposition method, the quantized wavelet coefficients in the multi-level dyadic subband decomposition is represented by three 1-D sequences of small alphabets.

#### 4. SIMULATION RESULTS

The proposed image coding scheme can be constructed in the following steps. (1) Generate a given level dyadic wavelet transform of an input image, (2) Linearly quantize the wavelet coefficients with a proper dead-zone, (3) Generate a quadtree code and a 1-D integer sequence with the quadtree decomposition method, (4) Convert the integer sequence into an L sequence and an R sequence with the data composition method, and (5) Compress the quadtree code, the L sequence, and the R sequence with an adaptive arithmetic code.

Both the Lena image and the Barbara image were compressed with the proposed algorithm, the EZW algorithm, and the SPIHT algorithm. Our simulation results indicate that the proposed coding algorithm is superior to EZW and competitive with SPIHT, as shown in Figure 1 for the Lena image and Figure 2 for the Barbara image. The comparison between the proposed coding scheme and our former 1-D coding scheme for the Lena image is shown in Figure 3.

#### 5. CONCLUDING REMARKS

We have presented in this paper a new wavelet image coding scheme, in which we use the quadtree code to exploit the 2-D nature of the quantized wavelet zeros in the dyadic decomposition, and with the data decomposition method, the integer sequence for remaining zeros and non-zero integers is converted to L and R sequences. The proposed wavelet image coding scheme is both computationally and conceptually simple. However, our simulation results have indicated that the coding efficiency of the proposed wavelet image coding scheme is competitive with those of the best image coders in the literature such as the SPIHT algorithm.

#### 6. ACKNOWLEDGEMENTS

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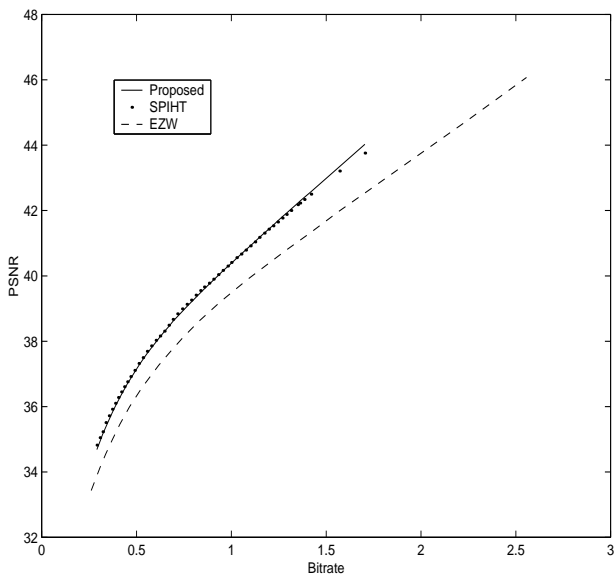


Figure 1: Coding results for the Lena image

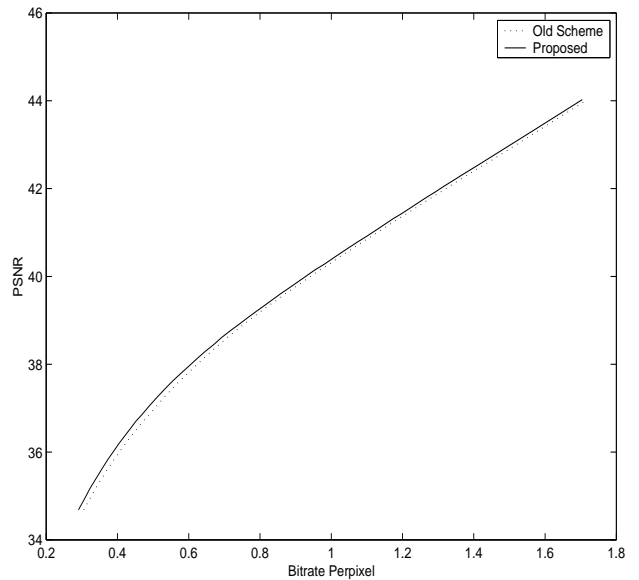


Figure 3: Comparison for the Lena image

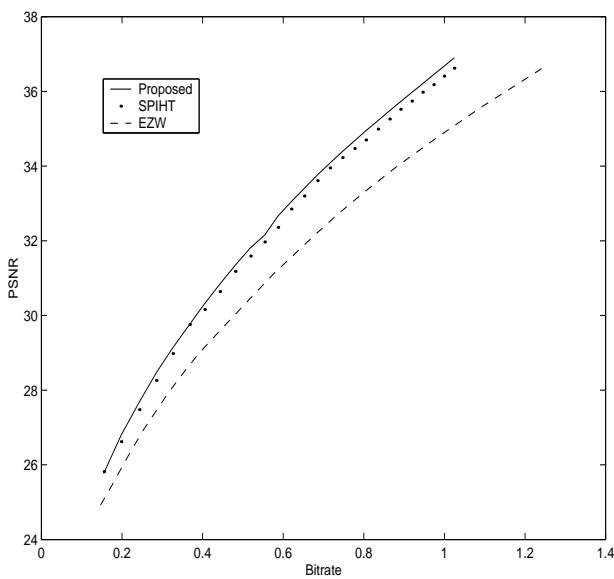


Figure 2: Coding results for the Barbara image